Oscillations

1 Introduction

Oscillation

Oscillation is a *repetitive* or cyclical *variation* of a quantity. Vibration is subset of oscillation involving mechanical systems.

Restoring Force

An oscillating body must have a *restoring force* that will always try to bring it back to its equilibrium position whenever it is displaced from that position.

Damped or Undamped

A *damped* oscillator is one which experiences resistive forces during oscillation while an undamped oscillator does not. The effect of damping is that the oscillations will die out over time.

Free or Forced

A *free* oscillator is under the influence of a restoring force and may be damped or undamped. No other forces act on the oscillator. A *forced* oscillator is *continuously* being driven by an external periodic force in addition to the restoring force.

Harmonic or Anharmonic

A *harmonic* oscillator is one whose *restoring force* is proportional to the displacement from its equilibrium. If the restoring force is not proportional to the displacement, then it is *anharmonic*.

Examples of Free Oscillations

A pendulum bob swinging to and fro has a cyclical variation of *position*.

A fluctuating electric *voltage* in our power supply.

The vibration of a plucked guitar string.

2 Simple Harmonic Motion (SHM)

Definition

SHM is an oscillatory motion that is *free*, *harmonic* and *undamped*. The resulting motion is characterised and defined as follows:

> SHM is an oscillatory or *periodic* motion whereby the acceleration is *proportional* to the displacement from the equilibrium position and always *towards the equilibrium* position.

Mathematically, the definition is:

$$\vec{a} = -\omega^2 \vec{x} \quad \text{---- (Eq. 1.1)}$$

where $\vec{a}$ is the acceleration, $\vec{x}$ is the displacement and ω is a constant depending on the characteristics of the system.

The negative sign reflects the opposite directions of $\vec{a}$ & $\vec{x}$.

Simple harmonic motion is a *periodic* motion whereby the acceleration is *proportional* and *opposite* to the displacement from the equilibrium position.

$\vec{a} = -\omega^2 \vec{x}$

or just magnitudes:
$a = \omega^2 x$

Example - Mass-spring Oscillator

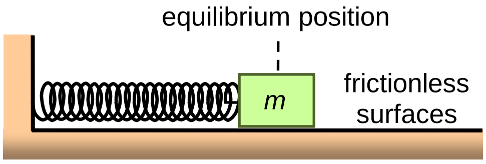


Fig. 2.1

Assume the spring can exert both pushing and pulling forces and this elastic force is given by Hooke's law $F_e = kx$ (magnitudes) when the mass has a displacement x from the equilibrium position.

When the mass is displaced to the right, the spring pulls it to the left. When the mass is displaced to the left, the spring pushes it to the right. Thus the directions of the elastic force and the displacement are always opposite and we can represent the relative directions by writing $\vec{F}_e = -k\vec{x}$. The elastic force always tries to restore the mass back to the equilibrium position. Hence the *restoring force* for this system is the elastic force.

Vertically, the normal force cancels the weight. Horizontally the elastic force is the only force so $\vec{F}_e = \vec{F}_{net}$. Therefore $-k\vec{x} = m\vec{a}$ and it can be re-written as $\vec{a} = -\omega^2\vec{x}$ where $\omega^2 = \dfrac{k}{m}$. In other words, the mass-spring set-up fulfils the defining condition for SHM and so we expect that when the mass is given an initial displacement and let go, the subsequent oscillatory motion would be classified as SHM.

Example - Swinging Pendulum

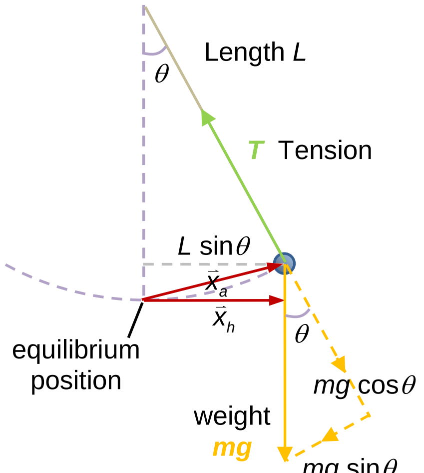


Fig. 2.2

Only 2 forces on bob - T & mg

$T = mg\cos\theta$
$F_{net} = mg\sin\theta$ --- (A)

$\vec{x}_h$ is bob's *horizontal* displacement.

$|\vec{x}_h| = L\sin\theta$ --- (B)

Replacing F_{net} by ma in (A) and using $\sin\theta = x_h/L$ from (B):

$$ma = mg\left(\frac{x_h}{L}\right)$$

$$a = \left(\frac{g}{L}\right)x \quad \text{where } \omega^2 = \left(\frac{g}{L}\right)$$

Hence $a = \omega^2 x$ again where $\omega^2 = g/L$. When θ gets increasingly smaller, the bob's *actual* displacement $\vec{x}_a$ would become increasingly closer to the horizontal displacement $\vec{x}_h$ (see Fig. 2.2). The direction of $\vec{a}$ is given by direction of $mg\sin\theta$ component which is opposite to that of displacement when θ is small. Thus *the smaller the angle* θ, the closer the pendulum's motion matches SHM.

In this case, the restoring force is provided by a component of the weight. In general, any oscillator whose *restoring force* or *net force* or *acceleration* can be shown to be directly proportional and opposite to the displacement from the equilibrium position must be a simple harmonic oscillator.

We can also see that $\omega^2 = k/m$ and $\omega^2 = g/L$ are the constants of proportionality in the defining equation $\vec{a} = -\omega^2\vec{x}$. In general ω is dependent on quantities which are characteristics of the set-ups like spring constant k, mass m, gravitational field strength g and length L.

Any motion that fits the equation $\vec{a} = -\omega^2\vec{x}$ is SHM, i.e.
1 $a \propto x$
2 $\vec{a}$ opposite to $\vec{x}$

Different set-ups involve different forces providing the restoring force leading to ω dependent on different system properties.

3 SHM - Describing Motion

The SHM equation $\vec{a} = -\omega^2\vec{x}$ can be written as $\dfrac{d^2\vec{x}}{dt^2} = -\omega^2\vec{x}$. The 2nd equation is known as a differential equation which like a quadratic equation has solutions satisfying the equation. In this case, the solutions are sinusoidal functions like $\vec{x} = x_0\sin(\omega t)$ and $\vec{x} = x_0\cos(\omega t)$ where -ve and +ve values are opposite in directions. Solving differential equations is not in the syllabus so we will look at other ways of arriving at these solutions.

$\vec{x}$ - t Plot

If a pen is attached to an oscillating mass and its motion is recorded on a moving strip of paper, the resulting plot shows how the displacement varies with time:

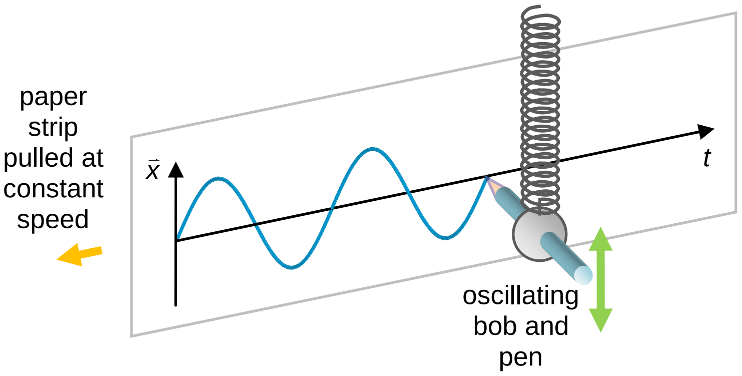


Fig. 3.1

It turns out that the $\vec{x}-t$ plot will be *sinusoidal* i.e. sine and all shifted sine plots. The plot will be described by $\vec{x} = x_0 \sin(\omega t)$ if the pen is only brought to touch the paper when the bob is passing the equilibrium position on its way up like in Fig. 3.1 and upward direction is chosen to be positive.

If the pen only starts plotting when it is at its highest point and going downwards, plot will be $\vec{x} = x_0 \cos(\omega t)$:

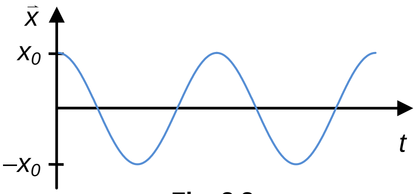


Fig. 3.2

If the pen starts plotting when it is slightly above the equilibrium position and going upwards, plot will be $\vec{x} = x_0 \sin(\omega t + \phi)$:

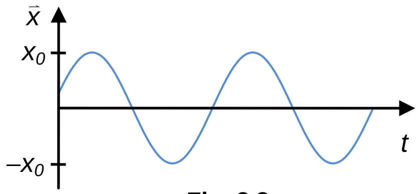


Fig. 3.3

When an oscillator fulfils the conditions for SHM, the resulting variation of displacement with time is *sinusoidal*. In fact, the reverse is also true i.e. a sinusoidal $\vec{x}-t$ implies SHM.

$\vec{v}$ - t & $\vec{a}$ - t Plots

From the above $\vec{x}-t$ plots, it is easy to obtain the corresponding $\vec{v}-t$ and $\vec{a}-t$ plots as $\vec{v} = d\vec{x}/dt$ and $\vec{a} = d^2\vec{x}/dt^2$:

$\vec{v} = -x_0\omega \sin(\omega t)$

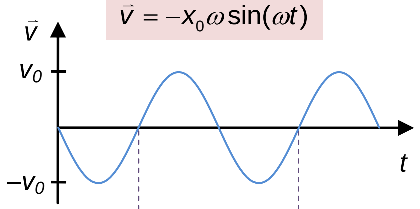


Fig. 3.4

$\vec{v} = x_0\omega \cos(\omega t + \phi)$

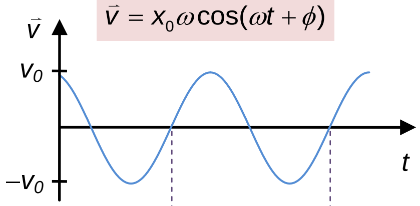


Fig. 3.5

$\vec{a} = -x_0\omega^2 \cos(\omega t)$

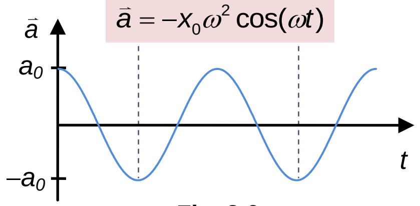


Fig. 3.6

$\vec{a} = -x_0\omega^2 \sin(\omega t + \phi)$

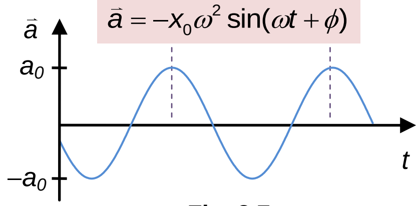


Fig. 3.7

The maximum magnitude on the vertical axis is called *amplitude*. Hence the amplitude of $\vec{x}-t$ plot is x_0, the amplitude of $\vec{v}-t$ plot is v_0 (= ωx_0) and the amplitude of $\vec{a}-t$ plot is a_o (= $\omega^2 x_0$ = ωv_0).

If displacement amplitude = x_0 then

1. velocity amplitude $v_0 = \omega x_0$
2. acceleration amplitude $a_0 = \omega^2 x_0 = \omega v_0$

Phase Angles and Phase Difference

Phase Angle

In sinusoidal functions, the quantity ωt or $(\omega t + \phi)$ is a *phase angle* measured in radians or degrees. The Moon goes through a cycle of different *phases* in 28 days. Similarly, a simple harmonic oscillator goes through a cycle of different displacements and velocities as the phase angle increases with t.

In the mathematical sinusoidal functions - $\sin\theta$ or $\cos\theta$ - as θ changes by 2π radians or 360°, the functions' values go through one cycle of change. These functions only operate on angles. Therefore sin(2 s) or cos(10 cm) are meaningless. In order to have the functions vary with time t instead of angle θ, we need to 'map' or 'transform' the t variable to the θ variable using a *conversion factor* just like an exchange rate from one currency to another. This is done based on the following:

1. θ should be proportional to $t \Rightarrow \theta = ct$
2. one cycle in angle is 2π while one cycle in time is a period T

   $\Rightarrow 2\pi = cT$

   Sub $c = \dfrac{2\pi}{T}$ from (2) into (1)

$$\theta = \frac{2\pi}{T}t$$ where conversion factor c is $\dfrac{2\pi}{T}$

Earlier, the conversion factor was also found experimentally to be ω which depends on the properties of the oscillator system. Frequency f is defined to be $1/T$. In this case $\omega = 2\pi/T$ is called *angular frequency* (note that it is called *angular velocity* in Circular Motion)

The phase angle of a sinusoidal function can be written in terms of variable quantities which are not angles by using appropriate conversion factors.

The conversion factor is $\omega = \dfrac{2\pi}{T}$ and called the *angular frequency*.

Phase Difference

In $y_2 = \sin(\theta + 90°)$, when θ is zero, y_2's value is that of $y_1 = \sin(\theta)$ when $\theta = 90°$ and as θ increases, the value of y_2 will take on values of y_1 from 90° onwards i.e. the plot of y_2 is obtained by shifting y_1 to the left by 90°. After shifting, the resulting plot is equivalent to $y = \cos(\theta)$ as shown in Fig. 3.8.

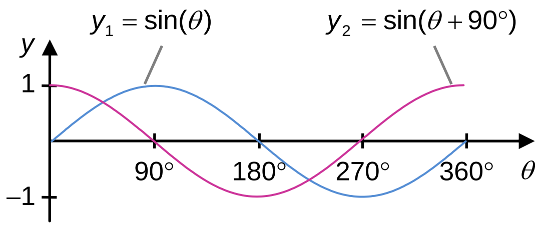


Fig. 3.8

In general, if angle ϕ is *added* to the phase angle of a sinusoidal function, the effect is to shift the function to the *left* by ϕ on the horizontal axis (minus sign corresponds to shifting to the right).

Comparing $y_2 = \cos(\theta) = \sin(\theta + 90°)$ and $y_1 = \sin(\theta)$ we can say that the phase angle of y_2 *leads* the phase angle of y_1 by 90°. When we are not interested in which phase angle is leading then we will just look at the absolute value of the difference in phase angles or *phase difference* in short. In this case, the phase difference ϕ is 90°.

Phase difference ϕ is thus the absolute value of the difference in phase angles of 2 sinusoidal functions and it can be found by looking at the amount of relative shift in terms of angle. In the context of 2 oscillators, ϕ reflects the *difference* in their positions and motions(velocity and acceleration).

Phase difference ϕ is the absolute value of the difference in phase angles of 2 sinusoidal functions, found by looking at the amount of relative shift in terms of angle.

ϕ reflects the *difference* in positions and motions of 2 oscillators.

If the horizontal axis is t instead of θ, the relative shift in terms of angle can still be easily found by remembering that T corresponds to 360° or 2π. In Fig. 3.9, the relative shift is $T/8$ and so ϕ must be 360°/8 or 45°.

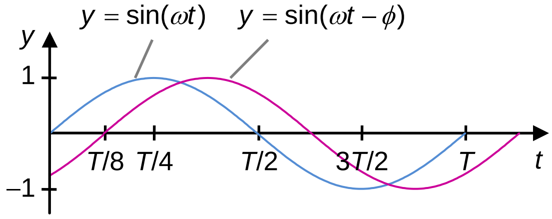


Fig. 3.9

To arrive at a formula, use the fact that phase angle is related to t by $\theta = \frac{2\pi}{T}t$, thus difference in phase angles or $\phi = \Delta\theta = \frac{2\pi}{T}\Delta t$.

Phase difference due to Δt is given by $\phi = \frac{2\pi}{T}\Delta t$ or

$\phi = \frac{360°}{T}\Delta t$

$\vec{a}$ - $\vec{x}$ and $\vec{v}$ - $\vec{x}$ Plots

Based on the relation $\vec{a} = -\omega^2\vec{x}$, the $\vec{a} - \vec{x}$ plot is as shown in Fig. 3.10 where +/– signs indicate directions.

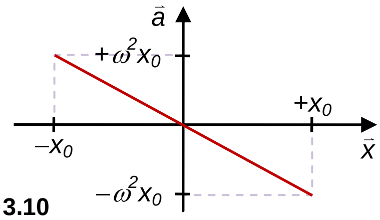


Fig. 3.10

As shown in previous section, the general $\vec{x} - t$ function for SHM is

$$\vec{x} = x_0 \sin(\omega t + \phi) \quad \text{---- (1)}$$

then $\quad \vec{v} = x_0\omega\cos(\omega t + \phi) \quad$ ---- (2)

$$\vec{v} = \pm x_0\omega\sqrt{1 - \sin^2(\omega t + \phi)} \quad \text{as} \quad \sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) = 1$$

$$\vec{v} = \pm\omega\sqrt{x_0^2 - x_0^2\sin^2(\omega t + \phi)}$$

$$\boxed{\vec{v} = \pm\omega\sqrt{x_0^2 - \vec{x}^2}} \quad \text{using } \vec{x}^2 = x_0^2\sin^2(\omega t + \phi) \text{ from (1)}$$

We have eliminated the t variable from equations (1) & (2) to arrive at a formula relating variables $\vec{v}$ and $\vec{x}$ instead.

Using $\vec{v} = \pm\omega\sqrt{x_0^2 - \vec{x}^2}$, we can then get $\vec{v} - \vec{x}$ plot as shown in Fig. 3.11.

The variation of $\vec{v}$ with $\vec{x}$ is $\vec{v} = \pm\omega\sqrt{x_0^2 - \vec{x}^2}$

'±' reflects the fact that for each $\vec{x}$ there are two possible $\vec{v}$ directions as shown in Fig. 3.11.

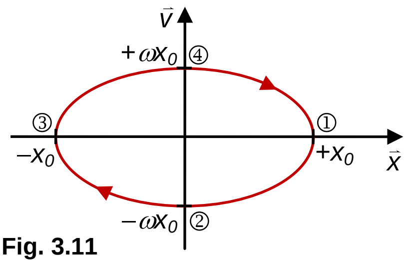


Fig. 3.11

For $\vec{x} = x_0\cos(\omega t)$ the plot goes from ①→②→③→④→① (Fig. 3.11 & 3.12).

For $\vec{x} = -x_0\cos(\omega t)$ the plot goes from ③→④→①→②→③.

For $\vec{x} = x_0\sin(\omega t)$ the plot goes from ④→①→②→③→④.

For $\vec{x} = -x_0\sin(\omega t)$ the plot goes from ②→③→④→①→②.

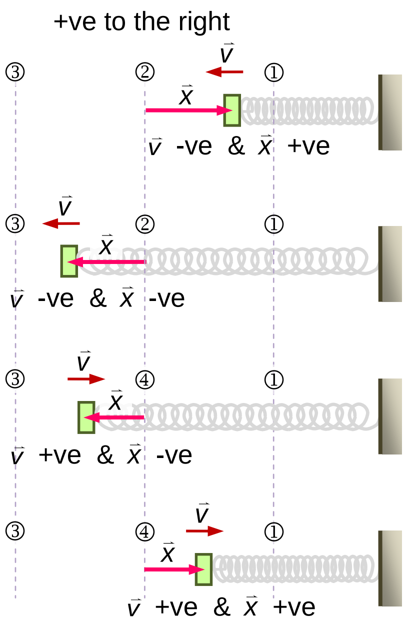
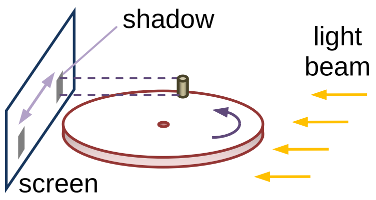


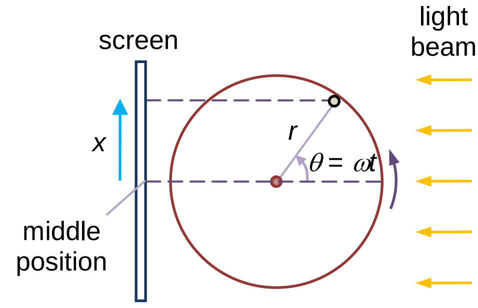
Fig. 3.12

Uniform Circular Motion and SHM

Shadow moves in SHM horizontally as peg on turntable undergoes uniform circular motion.



Side view



Plan view

Fig. 3.13

The shadow's displacement from the middle position is $x = r\sin(\omega t)$ which is *sinusoidal* and thus the shadow's motion is SHM.

The projection or shadow of an object in *uniform circular* motion is in *simple harmonic* motion.

4 SHM - Energies

E - *t* Plots

Plotting starts at $x = 0$

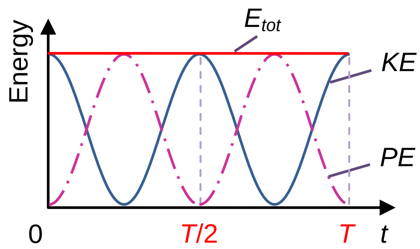


Fig. 4.1a

Plotting starts at $x = \pm x_0$

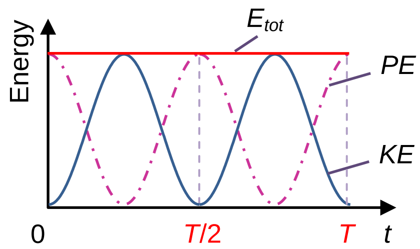


Fig. 4.1b

E-t plots for SHM show:

1 to and fro switching between KE and PE.

2 there are 2 cycles of variation in KE and PE for one cycle of variation in displacement.

3 total energy is constant as resistive forces are absent.

Note that oscillation period for *displacement* is *T* but for *KE and PE* is *T/2*.

Why is the total energy constant? A simple harmonic oscillator by definition can only have a restoring force and no resistive forces or other external forces. Hence no energy is lost by the oscillator due to resistive forces and no work is done on it by other external forces. However $E_{tot} = KE + PE$ is continuously switching between totally kinetic and totally potential.

$$E_{tot} = KE_{max} = \frac{1}{2}mv_0^2 = \frac{1}{2}m(\omega x_0)^2$$

To see how *KE* and *PE* vary with *t*, write *v* in time varying form:

oscillator at $x = 0$ when $t = 0$	oscillator at $\pm x_0$ when $t = 0$
$\vec{v} = x_0\omega\cos(\omega t)$	$\vec{v} = x_0\omega\sin(\omega t)$
$KE_t = \frac{1}{2}mv^2 = \frac{1}{2}m[x_0\omega\cos(\omega t)]^2$ $KE_t = \frac{1}{2}mx_0^2\omega^2\cos^2(\omega t)$	$KE_t = \frac{1}{2}mv^2 = \frac{1}{2}m[x_0\omega\sin(\omega t)]^2$ $KE_t = \frac{1}{2}mx_0^2\omega^2\sin^2(\omega t)$
$PE_t = E_{tot} - \frac{1}{2}mv^2$ $PE_t = \frac{1}{2}mx_0^2\omega^2 - \frac{1}{2}mx_0^2\omega^2\cos^2(\omega t)$ $PE_t = \frac{1}{2}mx_0^2\omega^2\left(1-\cos^2\omega t\right)$ $PE_t = \frac{1}{2}mx_0^2\omega^2\sin^2\omega t$	$PE_t = E_{tot} - \frac{1}{2}mv^2$ $PE_t = \frac{1}{2}mx_0^2\omega^2 - \frac{1}{2}mx_0^2\omega^2\sin^2(\omega t)$ $PE_t = \frac{1}{2}mx_0^2\omega^2\left(1-\sin^2\omega t\right)$ $PE_t = \frac{1}{2}mx_0^2\omega^2\cos^2\omega t$

To express *KE* in terms of *t*, use the *v-t* expression in $KE = ½mv^2$.

To express *PE* in terms of *t*, use

1 $E_{tot} = KE_{max}$

2 $PE = E_{tot} - KE$

E - $\vec{x}$ Plots

To see how *KE* and *PE* vary with *x*, write *v* in terms of *x*.

Use $\vec{v} = \pm\omega\sqrt{x_0^2 - \vec{x}^2}$ as derived earlier.

$$KE_x = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_0^2 - \vec{x}^2)$$

and

$$PE_x = E_{tot} - \frac{1}{2}mv^2$$

$$PE_x = \frac{1}{2}mx_0^2\omega^2 - \frac{1}{2}m\omega^2(x_0^2 - \vec{x}^2)$$

$$PE_x = \frac{1}{2}m\omega^2\vec{x}^2$$

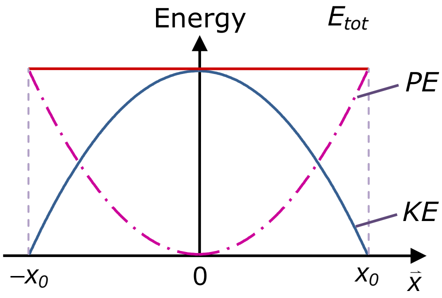


Fig. 4.2

To express *KE* in terms of *x*, use $\vec{v} = \pm\omega\sqrt{x_0^2 - \vec{x}^2}$ in $KE = ½mv^2$.

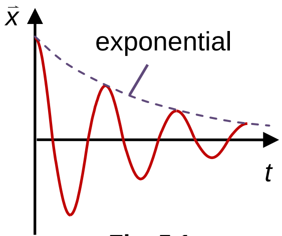
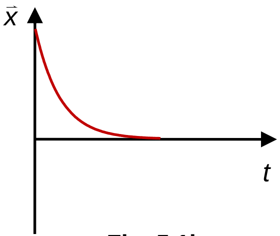
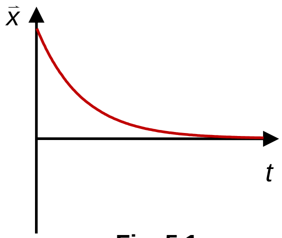
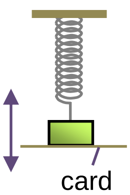
To express *PE* in terms of *x*, use $PE = E_{tot} - KE$

5 Damping

When resistive forces such as friction and viscous force are allowed to act on a simple harmonic oscillator we say that it is being *damped*. The effect of damping is negative work done on the oscillator leading to energy loss. The oscillator's motion is then *no longer simple harmonic*. However, resistive forces are mostly impossible to remove completely, so if they are small we can still treat the oscillator as close to simple harmonic.

A *damped* oscillator is one which experiences resistive forces leading to loss of energy as heat.

Damping is classified into three categories.

Under/light damping	Critical damping	Over/heavy damping
 **Fig. 5.1a**	 **Fig. 5.1b**	 **Fig. 5.1c**
Oscillations have gradually decreasing amplitude. Period increases with increased damping.	Displaced oscillator returns to equilibrium in shortest possible time with no oscillation.	Displaced oscillator returns to equilibrium longer than in critical damping with no oscillation.
Damping is less than critical.	A precise amount of damping marks the separation of light and heavy damping.	Damping is greater than critical.
Example  Oscillator with card attached encounters non-negligible air resistance.	Example Car's suspension system fitted with damper to minimise oscillations yet not over damped till too stiff for comfort. Critical damping is used in machines, buildings, bridges to remove vibrations.	Example  Auto-closing door with damper set to heavy damping so that door closes gently.

Three kinds of damping:

1 *Under damping* where oscillations have exponential drop in amplitude.

2 *Critical damping* where initial displacement decreases to zero in shortest possible time.

3 *Over damping* where excessive resistive force causes displacement to drop to zero in longer time than for critical damping.

6 Forced Oscillations

Natural Frequency

Every simple harmonic oscillator when disturbed will oscillate at a characteristic frequency known as *natural frequency*. For example, the mass spring system has $\omega^2 = \frac{k}{m}$ and since $\omega = 2\pi f$, its natural frequency is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ where k and m are the spring constant and mass respectively. Different systems thus have different characteristic natural frequencies.

Natural frequency of a simple harmonic oscillator is a *characteristic* frequency of oscillation dependent on system's properties.

Forced Oscillations & Resonance

Forced oscillations refer to the application of a periodic driving force to force an oscillator to oscillate at the frequency of the driving force. If a simple harmonic oscillator is subjected to an external periodic driving force, after a period of erratic motion it will eventually settle into a steady oscillation at the frequency of the driving force.

Forced oscillations refer to the application of a *periodic* driving force to force an oscillator to *oscillate at the frequency of the driving force*.

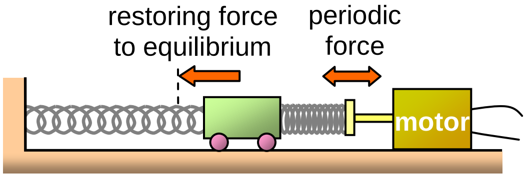


Fig. 6.1

Using the set-up in Fig. 6.1, the frequency of the driving force can be varied to investigate the oscillations at different frequencies. For this simple harmonic oscillator with a little damping, a series of driving frequencies f_D are chosen and for each f_D the *steady* state amplitude x_0 of the forced oscillation is measured and plotted. The resulting curve looks like 'underdamped 1' (Fig. 6.2) with a peak very close to the natural frequency f_N.

The variation of *steady amplitude* with driving frequency is shown in Fig. 6.2.

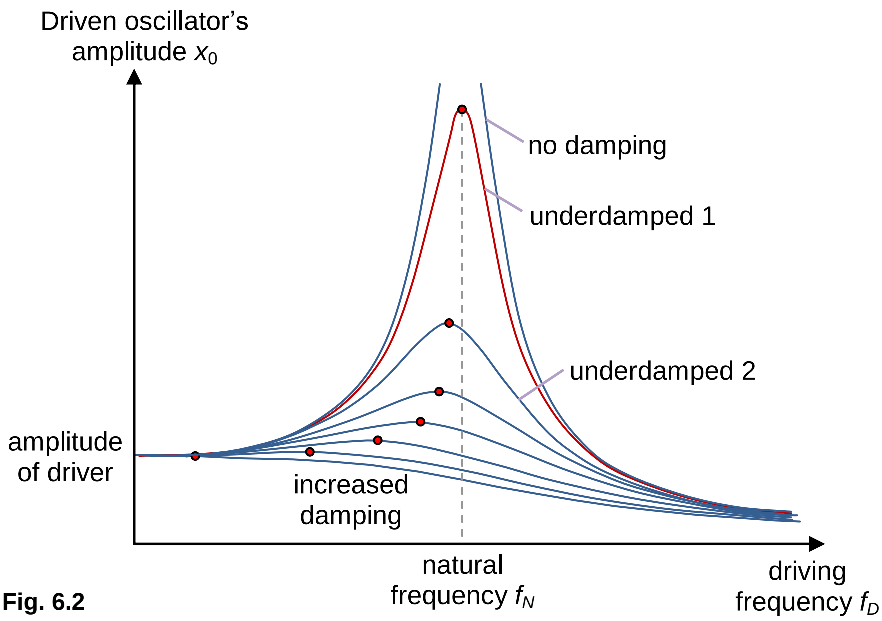


Fig. 6.2

To make sense of the curve, consider a child on a swing which is approximately a simple harmonic oscillator. After the first push, she will move forward and back with the natural frequency. If you push again just when she is swinging forward, you would do positive work and impart energy to her thus increasing her amplitude x_0. However if you push when she is swinging backwards you would do negative work and remove energy thus decreasing her amplitude.

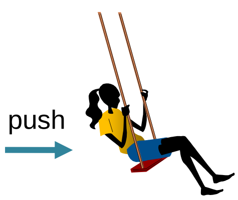


Fig. 6.3

It is easy to see that if your frequency of pushing matches the natural frequency the amplitude x_0 will get bigger and bigger but a limit will be reached. *Maximum amplitude* $x_{0\,\max}$ will be reached because damping will do more negative work with greater amplitude.

Now imagine that the frequency of pushing is halved i.e. one push for every 2 swings. There is now less energy per swing imparted to the child while the resistive forces are still continuously taking energy out of each swing. The final steady amplitude will be smaller than $x_{0\,\max}$.

Next imagine triple the original frequency i.e. 3 pushes per swing. Now some of the pushes will take place during the backward swing and so the final steady amplitude will again be smaller than $x_{0\,\max}$.

The driving force in the mass-spring system is actually more complicated than the pushes in the swing scenario. In general, when the driving frequency f_D doesn't match the natural frequency f_N, there will be times when the push and pull of the driver would end up impeding rather than helping the oscillation. Hence the *maximum steady amplitude* will only be achieved when driving frequency is near the natural frequency.

In theory, when there is no damping and $f_D = f_N$ the amplitude can go to infinity since there is only input of energy but no loss from the oscillator. In practice, if only a very small amount of damping is present and $f_D = f_N$, the amplitude can get so large that the oscillating system will break apart.

> The reaching of maximum amplitude by an oscillator
> when the frequency of an external periodic force
> is equal to the *natural frequency* is called *resonance*.
> During resonance, there is maximum rate of transfer of energy
> from the driver to the driven oscillator.

For a lightly damped simple harmonic oscillator, its natural frequency is progressively lowered with increasing damping.

Fig. 6.2 shows that if the amount of damping is increased,

1. the amplitude-frequency curve will be lowered
2. the peak will shift towards lower frequency &
3. the width of the spike is wider i.e. spike is less sharp.

Applications of Resonance

Undesirable Oscillation and Resonance

Metal sheets, wooden plank and more complicated structures like the frame of a car and casing of a washing machine all can vibrate somewhat when knocked or deformed a little. The vibrations caused by such onetime disturbance are not a big problem as they will die out sooner or later. A bigger problem is caused by a periodic driving force such as that of a motor. A motor has a core which can spin at variable frequency. The core's mass is rarely uniformly distributed around its axis of rotation thus leading to wobbling motion which causes its casing and any connected external parts to vibrate. As we saw earlier, when the frequency of the motor's spinning matches the resonant or natural frequency of the connected parts, the amplitude of vibration can become much bigger than that achieved by a single knock or displacement. Such big amplitudes can cause unwanted noise and breakage of the vibrating structures.

The reaching of maximum amplitude by an oscillator when the frequency of an external periodic force is equal to the *natural frequency* is called *resonance*. During resonance, there is maximum rate of transfer of energy from the driver to the driven oscillator.

Natural frequency for a lightly damped simple harmonic oscillator is progressively lowered with increasing damping.

3 effects of increased damping for x_0–f_D graph:

1 curve overall lowered

2 peak shifts to lower frequency

3 spike is less sharp

Resonance can cause unwanted problems such as noise and breakage.

Avoiding Oscillation and Resonance

What can be done to avoid the undesirable forced vibrations especially at resonance? One solution is to add *damping*. Firstly, it has the effect of reducing amplitudes at all frequencies. Secondly it can lower or shift the resonant frequency out of the frequency range of the driving force.

If we simplify the vibrating structure to a mass-spring system, we see that its natural frequency is determined by the spring constant and mass (see section 2 & start of section 6). Therefore, we can shift the resonant frequency out of the frequency range of the driving force by using different *materials* (different k) for the structure. Changing the *shape* of the structure has the effect of modifying the stiffness or elastic property as well as the mass distribution i.e. equivalent to changing k and m.

Broad ways to avoid resonance

1 use damping - reduce amplitudes and shift resonant freq.

2 change natural frequency - by changing material or shape

Damper

Buildings and bridges can also oscillate due to earthquake or wind. *Dampers* are often used to convert the vibrational energy to heat. Fig. 6.4 shows one type of damper. When connected between two pillars or parts that are vibrating, the piston will be pushed in and out of the cylinder of oil. The holes in the piston allow oil to flow through and the viscous drag converts the kinetic energy to heat. There are many other kinds of dampers but they all reduce the vibrations by converting kinetic energy to heat.

A damper

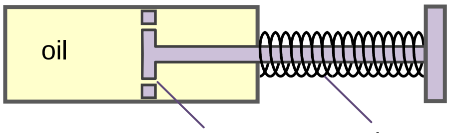


Fig. 6.4

Desirable Vibration and Resonance

Not all instances of resonance are bad. Resonance is an important part of how musical instruments produce musical notes and how we are able to produce different kinds of sound during speech.

Consider a string on a guitar or an air column in a flute. Both the string and air molecules in a column can be caused to vibrate. A string can be plucked or stuck. The air molecules can be set to vibrate when air flows past a sharp edge or reed of an instrument. A given length of string or air column has a number of resonant frequencies instead of just one in the case of a simple harmonic oscillator (details in topics of Waves and Superposition). The most dominant resonant frequency determines the overall pitch that we hear while the other weaker resonant frequencies and non-resonant frequencies determine the 'timbre' or sound character from an instrument. Thus by controlling the length of vibrating strings or air columns, we can produce the desired musical notes.

Resonance can also be desirable such as in producing musical notes and speech and electrical tuning circuits.

Antennae or receivers of electromagnetic(EM) waves coupled with appropriate electrical circuits have resonant frequencies that can be adjusted or tuned to resonate with the driving frequency of the EM waves. Thus the frequencies of the EM waves are received and converted to alternating voltages of the same frequencies. This is how radio broadcast is received and converted to electrical signals which in turn are converted to sound waves that we hear.