1 Introduction

This topic is called electro-magnetism because of the close link between electricity and magnetism. In fact, an electric current can produce a magnetic field which interacts with another current to give rise to motion. In reverse, motion of a conducting wire in a magnetic field can produce a current in that moving wire (next chapter Induction).

2 Magnetic Field

Similar to the concept of gravitational and electric field,

> *Magnetic field* is a *region of influence* in which a *magnetic pole* or *electric current* experiences a *magnetic force* from interaction with the field.

Magnetic field or 'B-field' can be produced by a magnet or current.

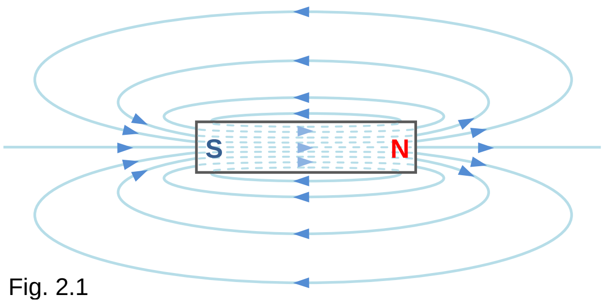

Fig. 2.1

The direction of B-field lines is from N to S pole outside a magnet but from S to N inside.

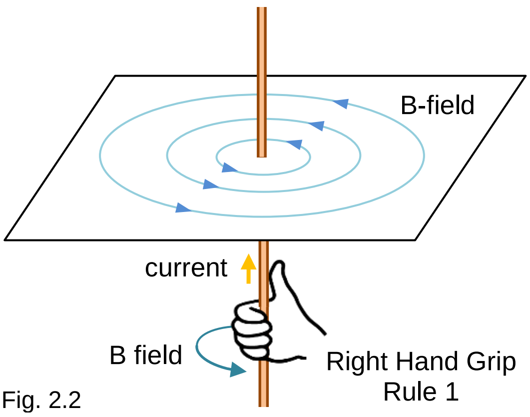

Fig. 2.2

Right Hand Grip Rule 1:

When right hand is gripping a conductor as shown, with the thumb pointing in direction of current, the fingers indicate the direction of B-field around the conductor.

Right Hand Grip Rule 2:

Wrap fingers around the coil such that fingers point in the direction of the currents. The thumb will then indicate the end that is N pole.

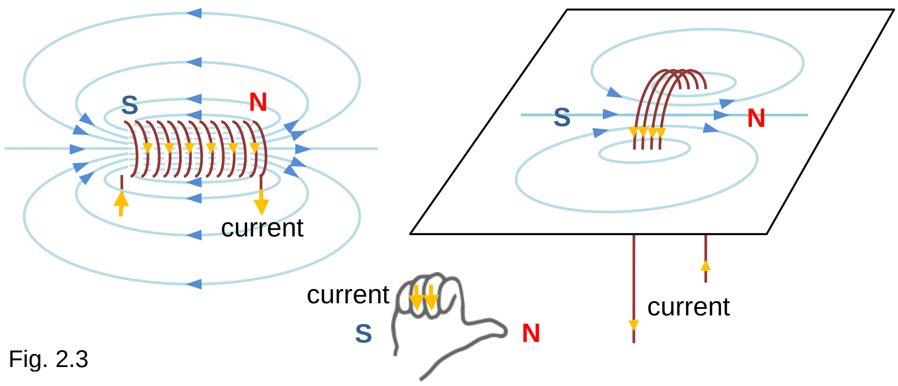

Fig. 2.3

Magnetic field is a *region of influence* in which a *magnetic pole* or *electric current* experiences a *magnetic force* from interaction with the field.

Direction of *magnetic field lines* or *flux lines* is from N to S pole *outside* a magnet or current loop, but from S to N *inside* the magnet or loop.

Right Hand Grip Rule 1 can be used to determine the direction of field lines around a straight current segment.

Right Hand Grip Rule 2 can be used to determine the poles for current loops or coils.

Magnetic Flux Density

The quantity that indicates the strength of a magnetic field is called *magnetic flux density* (symbol *B* or $\vec{B}$). It is a vector whose direction at a particular *point* is *tangential* to the curved field line at that point. The field lines are also known as *flux lines*. For a given area perpendicular to the flux lines, the more lines per unit area, i.e. greater flux density, the stronger the field.

$\vec{B}$ or *magnetic flux density* is visualised as the density of flux lines per unit perpendicular area.

Effect of Iron on $\vec{B}$

Iron and a few other materials have two very useful magnetic properties:

1. They channel or confine magnetic field due to the relative preference of field lines to pass through them.
2. They become strongly magnetised with an externally applied B-field.

Iron has 2 important properties:

1 It channels or confines B-field through it.
2 It becomes strongly magnetised with an externally applied B-field.

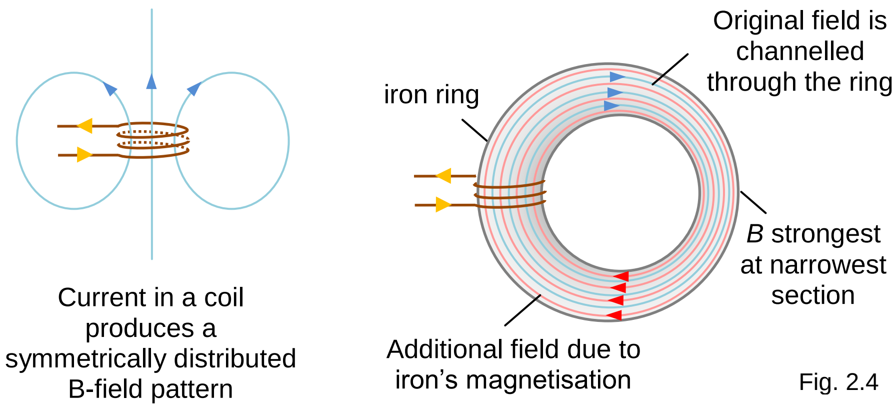

Fig. 2.4

The iron ring channels and confines the B-field within it, though sometimes with a little leakage. The iron's channelling of B-field and magnetisation leads to a much stronger magnetic field inside the coil and ring.

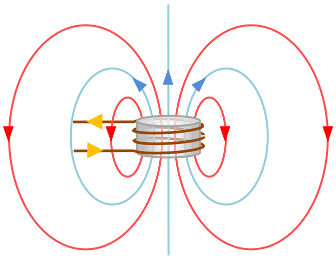

Fig. 2.5

When the coil is wrapped around a cylindrical iron core, the B field in and around the core can be increased by more than a thousand times.

When the current is turned off, the iron core quickly loses much of the magnetisation, thus the set-up is very useful as an electromagnet.

Origin of Magnetic Field

All magnetic fields are produced by current loops, even those from magnets and Earth. A typical atom has many electrons constantly moving around its nucleus. The combined movement of electrons may correspond to a current loop which gives rise to a tiny magnetic field. When all the tiny atomic magnets are aligned, the resultant field is strong enough to be noticed.

Similarly, our planet's field is believed to be due to electric currents in the conductive material of its core, powered by thermal convection currents. However, the details of the mechanism have not been fully understood.

Magnetic field points from *geographic* South Pole to *geographic* North Pole.

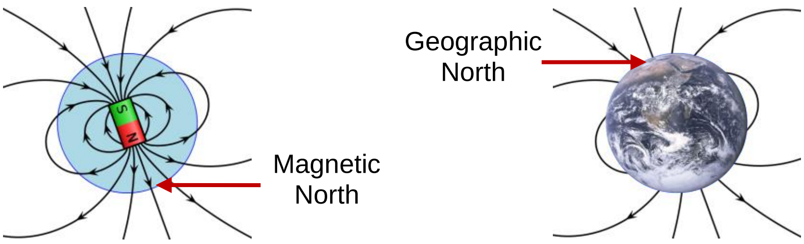

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Fig. 2.6

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3 Magnetic Force

Magnetic force arises from the interaction of a magnetic pole or current in a magnetic field. The nature of this force, specifically its direction and the factors affecting its magnitude will be the focus of this section.

Magnetic force arises from the interaction of a magnetic pole or current in a magnetic field.

Magnetic Forces on Poles

For magnetic poles, like poles repel while unlike poles attract.

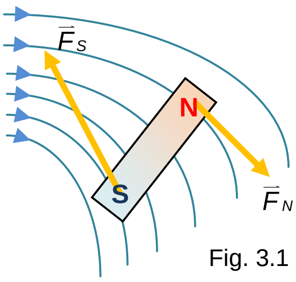


Fig. 3.1

The direction of magnetic force on N pole is in the same direction as that of the field lines at the pole while for S pole, the direction is opposite to that of the field lines.

$\vec{F}_S$ has a greater magnitude than $\vec{F}_N$ because the field experienced by the S pole is stronger.

Direction of magnetic force on N pole is the same as direction of field lines, while for S pole is opposite. Magnitude is greater when flux density is greater.

Magnetic Forces on Currents

There are two *basic* cases of interaction of a current with B-field:

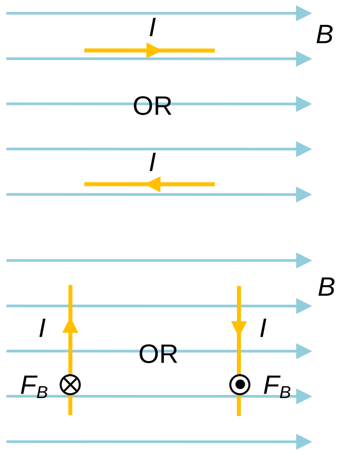


Fig. 3.2

When current I is parallel to the B-field, there is no magnetic force.

$F_B = 0$

When current I is perpendicular to the B-field, the magnetic force magnitude is given by

$F_B = BIL$

where L is length of conductor segment

⊗ indicates a direction into the page ⦿ indicates a direction out of the page

For magnitude of magnetic force F_B, 2 basic cases:

1 $\vec{B}$ // to I, no force.

2 $\vec{B}$ ⊥ to I, $F_B = BIL$

Fleming's Left Hand Rule (FLHR)

The nature of magnetic force is that its direction is always perpendicular to the current if it were produced at all. A rule called Fleming's *Left* Hand Rule(FLHR) has been devised to help us work out the direction of $\vec{F}_B$. Beware of confusing FLHR with Fleming's *Right* Hand Rule(FRHR) later.

Also note that FLHR does not explain the existence of $\vec{F}_B$. It is merely a method to figure out the direction of $\vec{F}_B$. What can be considered an explanation of the existence of a magnetic force $\vec{F}_B$ is 'the interaction of a current with a magnetic field' just like in the case of a 'mass/charge interacting with a gravitational/electric field' to produce a force.

Nature of magnetic force is that its direction is always perpendicular to the current if it were produced at all.

To use FLHR - keep the thumb, first and middle fingers *always* mutually perpendicular, then align with e.g. directions of $\vec{B}$ and I to find the direction of $\vec{F}_B$.

FLHR is used in situations where I interacts with $\vec{B}$ to give $\vec{F}_B$. It can be used to find the direction of I or $\vec{B}$ or $\vec{F}_B$ when the directions of the other two are known.

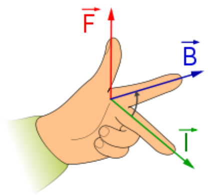
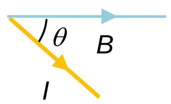


Fig. 3.3

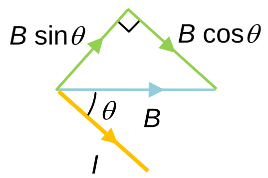
FLHR is used where I interacts with $\vec{B}$ to give $\vec{F}_B$. It can be used to find the direction of I or $\vec{B}$ or $\vec{F}_B$ when the directions of the other two are known.

When I and $\vec{B}$ are at Angle θ

(1) Given magnitudes B and I at angle θ:



(2) Resolve $\vec{B}$ into components parallel and perpendicular to I



(3)

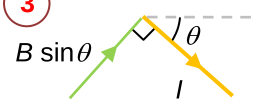


Fig. 3.4

$B\cos\theta$ is // to I, so no force. $B\sin\theta$ is $\perp$ to I, so magnitude of force is $F_B = (B\sin\theta)IL$

For direction of $\vec{F}_B$, FLHR gives a direction out of the page.

If I and $\vec{B}$ at angle θ, resolve $\vec{B}$ and then use $F_B = B_\perp IL$ and FLHR for magnitude and direction respectively.

Definitions - $\vec{B}$ & Tesla

While it is very useful to visualise magnetic flux density as the number of flux lines per unit area perpendicular to the lines, we need to be able to measure B for it to be more useful. Hence a formal definition is based on the relation $F_B = BIL$:

> *Magnetic flux density* is defined as
> the force *per* unit current *per* unit length of *straight* conductor
> placed *perpendicular* to a *uniform* magnetic field.

The SI unit for B is called Tesla and it is defined as:

> *Tesla* is defined as the amount of magnetic flux density,
> which produces *one newton* force *per metre* of straight conductor
> *per ampere* current,
> when that conductor is *perpendicular* to the *uniform* magnetic field.

Definitions: Magnetic flux density & Tesla (boxes to left)

Notice that in the definition of Tesla, it is incorrect to state 'per *unit* current' instead of 'per *ampere* current'. Also, it is necessary to state the circumstances of the measurement by specifying 'when that conductor is *perpendicular* to the *uniform* magnetic field'.

Measurement of B Using Current Balance

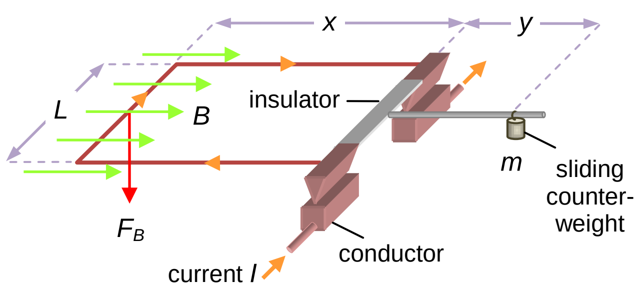


Fig. 3.5

A current balance, shown to the left, uses balancing of moments to find the magnetic force and then the magnetic flux density.

The leftmost segment of the conducting loop is subjected to a B-field as shown. As a result of the current in it flowing perpendicular to the B field, there is a downward force $F_B = BIL$.

When the loop is balanced by the counter-weight,

$$\begin{aligned} \text{anti-clockwise moment} &= \text{clockwise moment} \\ F_B x &= mgy \\ F_B &= mgy / x \\ BIL &= mgy / x \\ B &= mgy / ILx \end{aligned}$$

Hence, by measuring m, y, I, L and x, B can be determined.

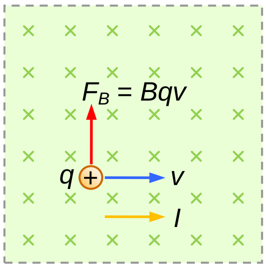
Magnetic Force on Moving Charge (not for H1 from here onwards)

A stream of moving charges or even a single moving charge makes up a current. The direction of current is the same as the direction of velocity for a positive charge whereas the direction of current is *opposite* that of the velocity for a *negative* charge.

From earlier, $F_B = BI_{\perp}L$ but $I = Q/t$ where charge Q flows in time t
So, $F_B = B\,(Q/t)\,L = BQ\,(L/t)$

$$F_B = BQv_{\perp}$$ where v is velocity of charge perpendicular to B

Magnitude of magnetic force on a charge moving in a magnetic field = $BQv_{\perp}$

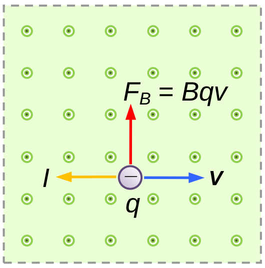


B into page

$F_B = BQv_{\perp}$ is used to find the magnitude of force.

FLHR is used to find the direction of F_B.

Fig. 3.6



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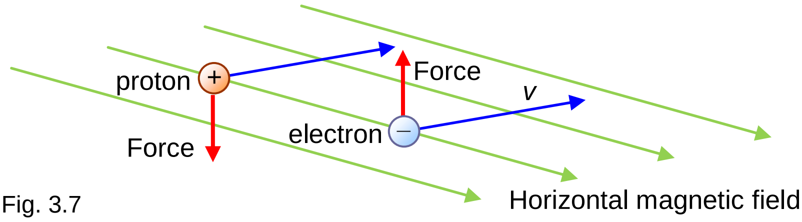


Fig. 3.7

If v and B are not perpendicular, resolve either v or B:

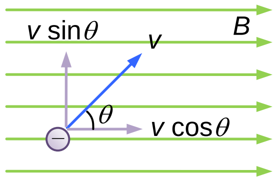
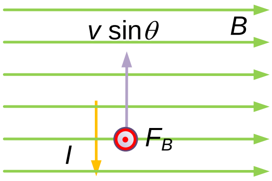


Fig. 3.8



Using component of v perpendicular to B,

$F_B = BQ\,(v \sin\theta)$

To apply FLHR, use only component of v perpendicular to B and identify the correct current direction.

4 Charged Particles in E and B Fields (not for H1)

An important note - we will focus on charged particles with tiny masses so that gravitational force is negligible compared to electric and magnetic forces.

This topic focuses on tiny particles with negligible gravitational force.

Inside Uniform E-Field

In a region of uniform E-field, the uniformity of the field means that the electric force will have constant *magnitude* and *direction* everywhere in the region. The result is parabolic trajectory for charged particles *inside* the field just like a projected ball on Earth's surface being subjected to a practically uniform gravitational field.

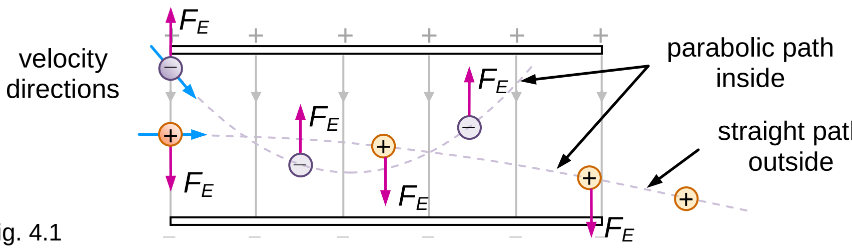


Fig. 4.1

Inside Uniform B-Field

A key feature of magnetic force is that it is *always perpendicular* to the current or charged particle's velocity if it is produced. Explained in the topic 'Circular Motion', a force that is *always* perpendicular to the velocity will only cause a change in *direction* of the velocity without changing its magnitude. Also if this force has a constant magnitude, there will be circular motion.

Magnetic force is *always perpendicular* to the current or charged particle's velocity if it is produced.

v Perpendicular to *B*

In Fig. 4.2a, assume a projected proton has velocity perpendicular to B-field. Here, the magnetic force is the only force acting (negligible gravitational force) and it becomes the net force for centripetal acceleration, hence

$$F_B = ma_c$$

$$BQv = m\frac{v^2}{r} \quad \text{or} \quad = mr\omega^2$$

Circular motion results when a charged particle has projected velocity *perpendicular* to a *uniform* B field.

To investigate factors affecting the radius of curvature *r*, write above as:

$$r = \frac{mv}{BQ} \qquad \text{---- (Eq. 4.1)}$$

Eq. 4.1 tells us that since a proton has *fixed* mass *m* and charge *Q*, the radius is affected by the speed *v* and the field *B*. If *B* is *uniform* and kept *constant* with respect to time, then a higher speed will result in a larger radius of curvature (see Fig. 4.2b).

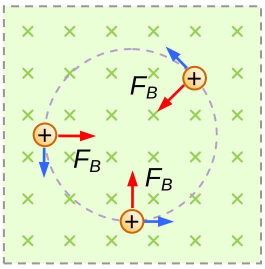


Fig. 4.2a

Charged particle projected from within the field

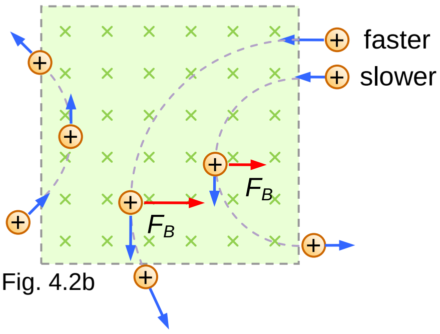


Fig. 4.2b

Charged particle projected from outside the field

In Fig. 4.2a, if *B* is kept uniform but its magnitude increasing, then F_B will be increasing and Eq. 4.1 tells us that *r* will be decreasing (recall *v* constant), thus leading to an inward spiral.

To investigate how the various factors affect the period of circular motion:

$$BQv = mr\omega^2 \qquad \text{but } v = r\omega$$

$$BQ = m\omega \qquad \text{where } \omega = \frac{2\pi}{T}$$

$$T = \frac{m2\pi}{BQ} \qquad \text{---- (Eq. 4.2)}$$

For fixed *m* and *Q*, we can see that period *T* is inversely proportional to *B*. Notice that in the working, if we did not substitute *v* for $r\omega$ we would get:

$$BQv = mr\left(\frac{2\pi}{T}\right)^2$$

$$T^2 = \frac{mr4\pi^2}{BQv} \qquad \text{---- (Eq. 4.3)}$$

If we are insensitive to the *constant* or *variable* nature of each quantity, we may wrongly conclude from Eq. 4.3 that $T^2 \propto (1/v)$, thus contradicting Eq. 4.2 which shows that *T* only depends on *B* but not *v*. What is wrong is that we cannot conclude $T^2 \propto (1/v)$ unless *B* and *r* are kept constant when *v* is varied, which turns out to be impossible according to Eq. 4.1

In considering how one quantity is affected by another quantity, it is important to be sensitive to the constant or variable nature of all quantities in equations.

v Component Along *B*

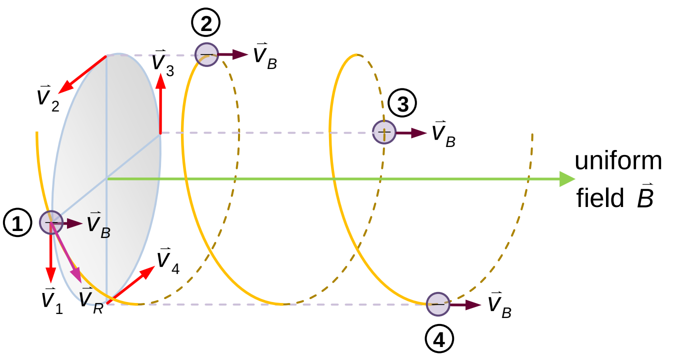


Fig. 4.3

Fig.4.3 shows a region of uniform magnetic field $\vec{B}$ perpendicular to a shaded plane. At position ①, a negatively charged particle has velocity $\vec{v}_R$. This velocity has a component $\vec{v}_B$ along the direction of the field and a component $\vec{v}_1$ perpendicular to the field. $\vec{v}_B$ does not interact with $\vec{B}$ since they are parallel. However, $\vec{v}_1$ interacts with $\vec{B}$ to produce a centripetal force and circular motion in the shaded plane. The combined circular motion and constant motion $\vec{v}_B$ towards the right results in a helical path as shown.

The particle at positions 2, 3 & 4 have velocity components $\vec{v}_2$, $\vec{v}_3$ & $\vec{v}_4$ in a the shaded plane where magnitudes $v_1 = v_2 = v_3 = v_4$. At any point along the helix, the particle has the same rightwards velocity $\vec{v}_B$.

When charged particle's velocity has a component parallel to the uniform B-field, its resultant path will be helical. Its motion is a combined circular motion and a constant velocity motion parallel to B-field.

Comparison of Charged Particles in *E* and *B* Fields

The comparison below is for particles with *constant* charge in *uniform* fields.

Uniform E-field	Uniform B-field
$\vec{F}_E$ has the same direction everywhere in field.	$\vec{F}_B$'s direction determined by the directions of velocity and field.
$\lvert\vec{F}_E\rvert$ depends on Q and E ($F_E = QE$).	$\lvert\vec{F}_B\rvert$ depends on B, Q, v and angle between $\vec{B}$ and $\vec{v}$ ($F_B = BQv \sin\theta$).
$\vec{F}_E$ changes the magnitude and eventually the direction of velocity.	$\vec{F}_B$ will not change the magnitude but *may* change the direction of velocity.
$\vec{F}_E$ does work and change particle's KE.	$\vec{F}_B$ does zero work and does not change particle's KE.

A contrast of $\vec{F}_E$ & $\vec{F}_B$ for a given charged particle in uniform E and B-fields is as shown on the left.

Perpendicular E and B Fields - Velocity Selector

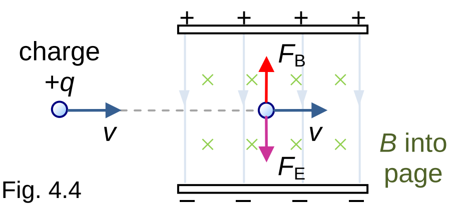


Fig. 4.4

$$F_E = F_B$$
$$qE = Bqv \quad \text{---- Eq. 4.4}$$

In a vacuum with perpendicular uniform E and B fields, it is possible to choose the directions and magnitudes of the fields such that F_E and F_B cancel out and the charged particle goes right through without any deflection.

Eq. 4.4 shows that changing the mass or charge of the particle would not change the outcome.

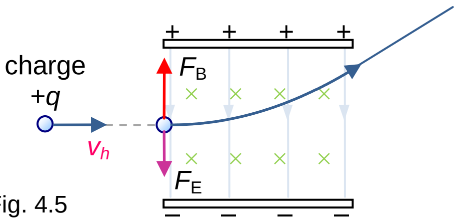


Fig. 4.5

In Fig. 4.4, if the particle were faster at speed $v_h > v$, then F_E would still be the same as before but F_B will be greater than before due to its dependence on velocity. This would lead to the outcome in Fig. 4.5.

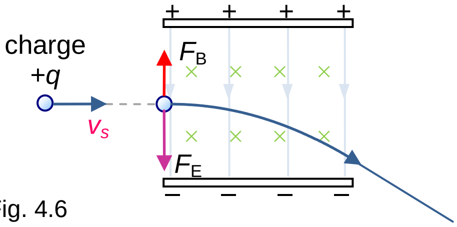


Fig. 4.6

In Fig. 4.4, if the particle were slower at speed $v_s < v$, then F_E would still be the same as before but F_B will be smaller than before due to its dependence on velocity. This would lead to the outcome in Fig. 4.6.

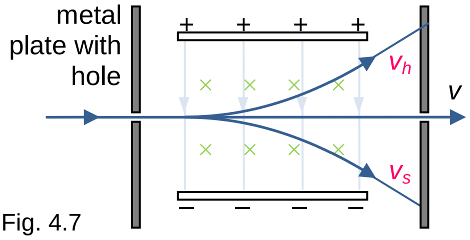


Fig. 4.7

If a whole bunch of positively charged particles were to enter the set-up in Fig. 4.7 with the following characteristics
- different masses
- different charge magnitudes
- different speeds

Which particles would emerge?

Again, in order to have no deflection,

$$F_E = F_B$$
$$QE = BQv$$
$$E = Bv$$
$$v = E/B$$

This derivation shows that charged particles of speed exactly = E/B would emerge while those faster or slower would deflect and get blocked by the second metal plate. The emerging particles can have different masses or charge magnitudes but they all have the same velocity.

The set-up in Fig. 4.7 is called a velocity selector since it selects particles of velocity = E/B to emerge. To select particles of a different velocity to emerge, we can control the ratio E/B. For instance, E can be adjusted by changing the p.d. across the parallel horizontal plates.

(Note that in Fig. 4.7, the set-up will not work if the B field direction were out of the page.)

Under the conditions:
- charged particles of different m, Q and v
- projected into a region such that directions of v, E and B are mutually perpendicular
- and the relative directions of E and B are chosen correctly e.g. in Fig. 4.7,

the particles that are not deflected will all have velocity $v = E/B$.