1 Electric Force

In 1700s, physicists found out experimentally that the force between charged particles depend on the amount of charge and the distance between them. In 1785, Coulomb published a paper stating his law:

> Every point charge exerts on another point charge an electric force
> that is proportional to the product of the charges but
> inversely proportional to the square of the separation and
> the force acts along a line joining the point charges.
> The force is repulsive for charges of the same sign and attractive for different signs.

The law is valid only for charges at rest. The *magnitude* of the force is given by $F_E = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}$ and the signs of Q_1 and Q_2 are dropped during substitution of values. ε is the *permittivity* of the medium that surrounds the charges. If the medium is vacuum (or free space), the permittivity is denoted by ε_0 = 8.85 × 10^{-12} F m^{-1}. Permittivity of air is approximately equal to that of vacuum.

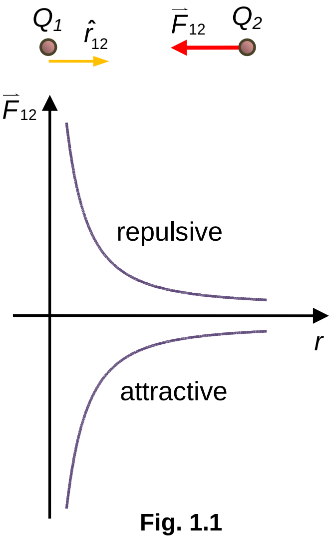

Fig. 1.1

The equation in vector form: $\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}\hat{r}_{12}$.

$\vec{F}_{12}$ is the force (vector) by charge Q_1 on charge Q_2 and $\hat{r}_{12}$ is the unit vector pointing from Q_1 to Q_2. The little cap on $\hat{r}_{12}$ indicates that $\hat{r}_{12}$ has a magnitude of one. During substitution of values, if Q_1 and Q_2 have the same sign, $\vec{F}_{12}$ = $+c\ \hat{r}_{12}$ where c is the magnitude of the force. The + sign thus indicates that $\vec{F}_{12}$ is in the same direction as $\hat{r}_{12}$ (repulsive). If Q_1 and Q_2 are of different sign, $\vec{F}_{12}$ = $-c\ \hat{r}_{12}$ whereby the – sign indicates that $\vec{F}_{12}$ is opposite in direction to $\hat{r}_{12}$ (attractive).

Spherical Charge Distribution

Just like a spherical mass can be considered as a point mass, a spherical charge distribution behaves like as a point charge at the centre of the sphere when interacting with an external charged body. Hence, the repulsive force in Fig. 1.2 and 1.3 are the same for the same r between Q_1 and Q_2.

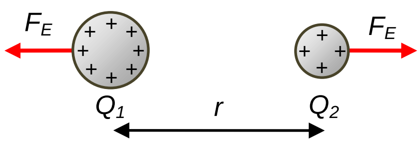

Fig. 1.2

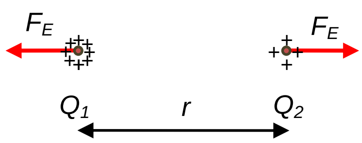

Fig. 1.3

Fig. 1.2 assumes that the spheres are not metallic or they are far apart. If they are metallic, the repulsion of the charges between the spheres will cause the charge distributions to become non-spherical as in Fig. 1.4. Then the effective distance between q_1 and q_2 will no longer be r. However, if r is much bigger than the radii, a small change in r is no big deal.

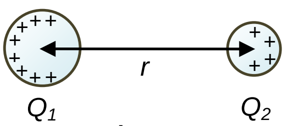

Fig. 1.4

Coulomb's law: Every point charge exerts on another point charge an electric force that is proportional to the product of the charges but inversely proportional to the square of the separation and the force acts along a line joining the point charges.

$$F_E = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}$$

The force is repulsive for charges of the same sign and attractive for different signs.

The law is valid only for charges at rest.

Spherical charge distributions behave like point charges.

2 Electric Field

An electric field is a *region* in which a charge experiences an electric force.

The *electric field strength* at a point is defined as the electric force per unit *positive* test charge at that point.

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$$\vec{E} = \frac{\vec{F}}{+q_{test}}$$

A test charge is a small amount of charge for the purpose of experiencing and measuring the force. It is kept small to avoid disturbing the charge distributions which are responsible for the field.

The test charge is *standardised* to be positive. The sign will determine the direction of the force experienced and hence the direction of the defined field.

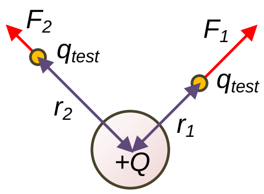
Based on the definition $\vec{E} = \frac{\vec{F}}{+q_{test}}$, electric field strength is a vector in the same direction as the electric force experienced by the positive test charge.

In Fig. 2.1a, a point charge +*Q* produces an electric field (E field) all around it. +*Q* can be regarded as the 'source' charge producing the E field. At two points at distances r_1 and r_2 from +*Q*, if a test charge $+q_{test}$ experiences a force of magnitude F_1 and F_2 respectively, the E field strength magnitudes and directions at those two points are shown in Fig. 2.1b.

Magnitude of E field strength due to point charge *Q* is given by

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

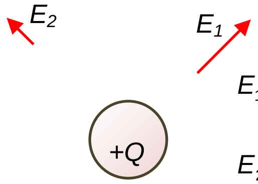
E field strength is the property of a point due to 'source' charge *Q*.



$$F_1 = \frac{Qq_{test}}{4\pi\varepsilon r_1^2}$$

$$F_2 = \frac{Qq_{test}}{4\pi\varepsilon r_2^2}$$

Fig. 2.1a



$$E_1 = \frac{Q}{4\pi\varepsilon r_1^2}$$

$$E_2 = \frac{Q}{4\pi\varepsilon r_2^2}$$

Fig. 2.1b

If the charge were negative –*Q*, the directions of forces and field strengths will be reversed.

E field Lines and Pattern

Just like gravitational field lines, E field lines are drawn around charges to indicate the strength and direction of the field. A closer the spacing of the lines indicate a stronger field. Arrows indicate the directions of the field.

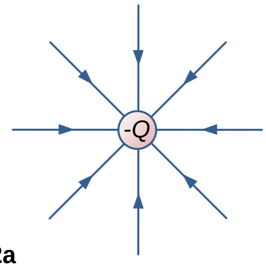


Fig. 2.2a

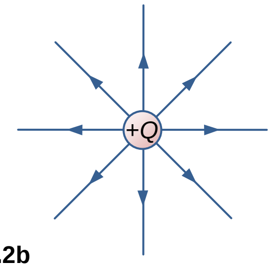


Fig. 2.2b

Field line arrows always point away from positive charges and towards negative charges.

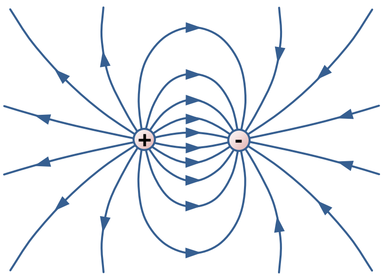


Fig. 2.3a

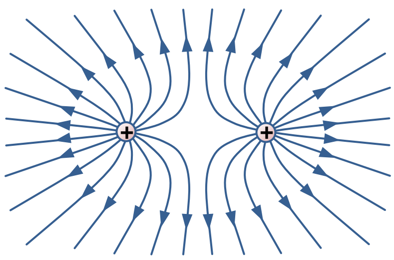


Fig. 2.3b

E field patterns due to 2 same magnitude point charges have symmetry about divider plane and the line joining them.

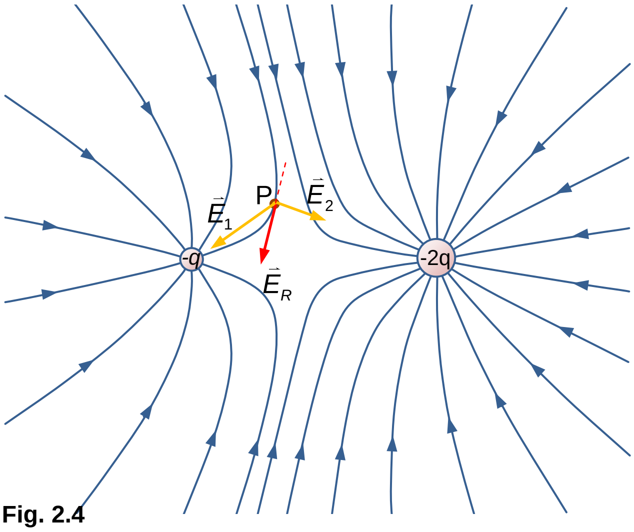


Fig. 2.4

The E field strength vector at any point is along the tangent to the field line at that point e.g. point P. The E field strengths at P due to *-q* and *-2q* are $\vec{E}_1$ and $\vec{E}_2$ respectively. The resultant field strength at P is $\vec{E}_R$ which is tangent to the field line at point P.

The E field strength vector at any point is along the tangent to the field line at that point.

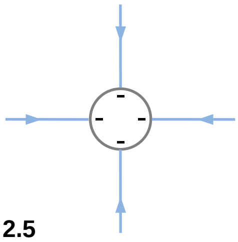


Fig. 2.5

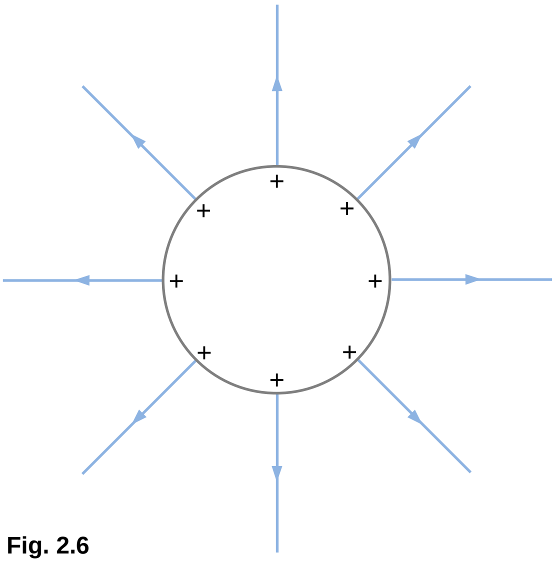


Fig. 2.6

E fields due to a hollow metal sphere charged negatively (Fig. 2.5) and positively (Fig. 2.6).

There is zero E field inside the spheres because of cancellation of the fields from individual charges.

Whenever there are many point charges, the resultant E field is always given by a *vector addition* of the individual E fields.

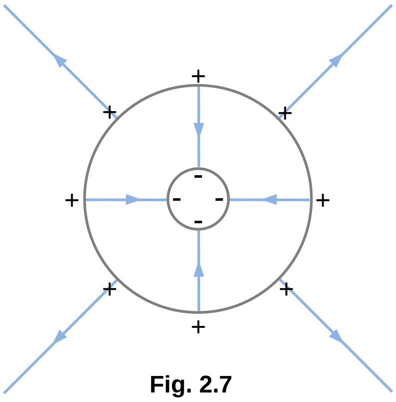


Fig. 2.7

When the negatively charged sphere is placed inside the positively charged sphere, the resulting E field is as shown in Fig. 2.7.

In Fig. 2.7, the field of Fig. 2.5 extends beyond the larger sphere, partially cancelling the field of Fig. 2.6, resulting in a weaker field outside the larger sphere.

In the space between the small and large spheres, there is only the E field due to the small sphere.

Similarly, charged objects whose charge distributions do not change when brought near to one another will produce a resultant field pattern that is a *vector addition* of their individual fields.

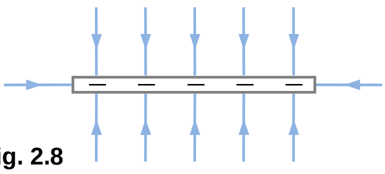


Fig. 2.8

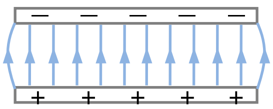


Fig. 2.9

A charged plate has E field pattern as shown in Fig. 2.8.

When 2 equal but oppositely charged metal plates are placed parallel to one another, the resultant field is shown in Fig. 2.9. There is reinforcement of field in the space between the plates and cancellation above and below the pair of plates.

Uniform E Field

In a uniform field, the *magnitude* and *direction* of the field strength are constant everywhere. Such fields are represented by *uniformly spaced* and *straight* field lines all pointing in the same direction.

To get a uniform field, a pair of parallel metal plates is charged by a battery (Fig. 2.10). The field between the plates is uniform except near the edges of the plates.

A charged particle placed inside a uniform field will experience a force of constant magnitude and direction anywhere in the field.

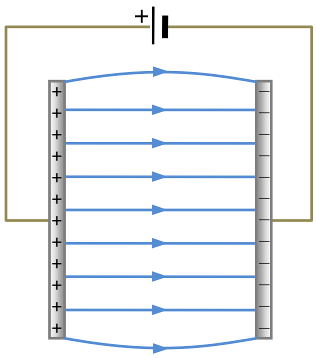


Fig. 2.10

A uniform field is one in which the *magnitude* and *direction* are constant everywhere.

3 Electric Potential Energy

Two point charges exerting electric forces on one another allow storage of electric PE between them. In Fig. 3.1,when the charge on the right is moved nearer to another of the same sign, positive work is done as the external force F_{ext} and displacement d are in the same direction. When this work is done *without changing the kinetic energy* of the system, the work gets stored as electric PE which can be released if the external forces are removed.

Fig. 3.1

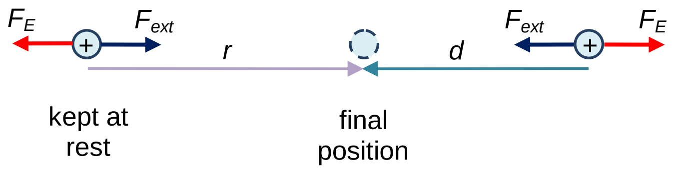
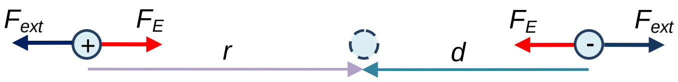


Fig. 3.2



In Fig. 3.2, if the charges are of different signs, moving the -ve charge nearer to the other without changing the system's KE will involve negative work which means removal of electric PE. Hence doing work on the system without changing its KE modifies its electric PE.

Taking the zero PE state to be when the charges are infinitely far apart:

> Electric PE between two point charges at a given separation is the work done by an *external* force to move one of the charges from infinity to that separation without changing the system's kinetic energy.

Electric PE between two point charges at a given separation is the work done by an *external* force to move one of the charges from infinity to that separation without changing the system's kinetic energy.

This definition leads to positive electric PE between charges of the same sign and negative electric PE between charges of different signs. The work done is not simply given by the product $F_{ext}\ d$ because the force is not constant. The calculation involves integration and the result is:

$$\text{EPE, } U_E = \frac{Q_1 Q_2}{4\pi\varepsilon r} \text{ between \textit{point} charges}$$

$U_E = \dfrac{Q_1 Q_2}{4\pi\varepsilon r}$ between *point* charges.

It is important when substituting values of Q to include the signs of the charges e.g. if Q_1 is +ve and Q_2 is -ve, U_E will be -ve. U_E is a scalar so the sign is an important part of its value.

In Fig. 3.2, three charges are equidistant from each other. The total U_E is a scalar addition of the PE between each pair of charges:

$$\text{Total } U_E = \frac{-q_1 q_2}{4\pi\varepsilon s} + \frac{q_2 q_3}{4\pi\varepsilon s} + \frac{-q_1 q_3}{4\pi\varepsilon s}$$

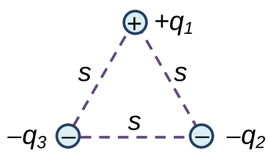


Fig. 3.3

4 Electric Potential

> Electric potential at a point is defined as
> the work done *per unit charge* by an external force
> to move the charge from infinity to that point.

Again, based on zero potential to be at infinity, the potential at a point which is at distance r from a *point* charge Q is:

$$\text{Electric potential, } V = \frac{Q}{4\pi\varepsilon r} \quad \text{due to } \textit{point} \text{ charge}$$

Just like electric PE, the sign of Q must be considered during substitution of values. Hence a +ve Q will cause the potential values around it to be +ve while a -ve Q will cause the potential values around it to be -ve.

The definition implies that V is a scalar measured in V(volt) or J C^{-1}. If there are a few charges around a point. The potential at that point will just be a scalar addition of the potentials due to all the surrounding charges.

Electric potential at a point is defined as the work done *per unit charge* by an external force to move the charge from infinity to that point.

$V = \frac{Q}{4\pi\varepsilon r}$ due to a *point* source charge Q.

Unit of V: V(volt) or J C^{-1}.

5 Relationships Between $\vec{F}_E$, $\vec{E}$, U_E , V

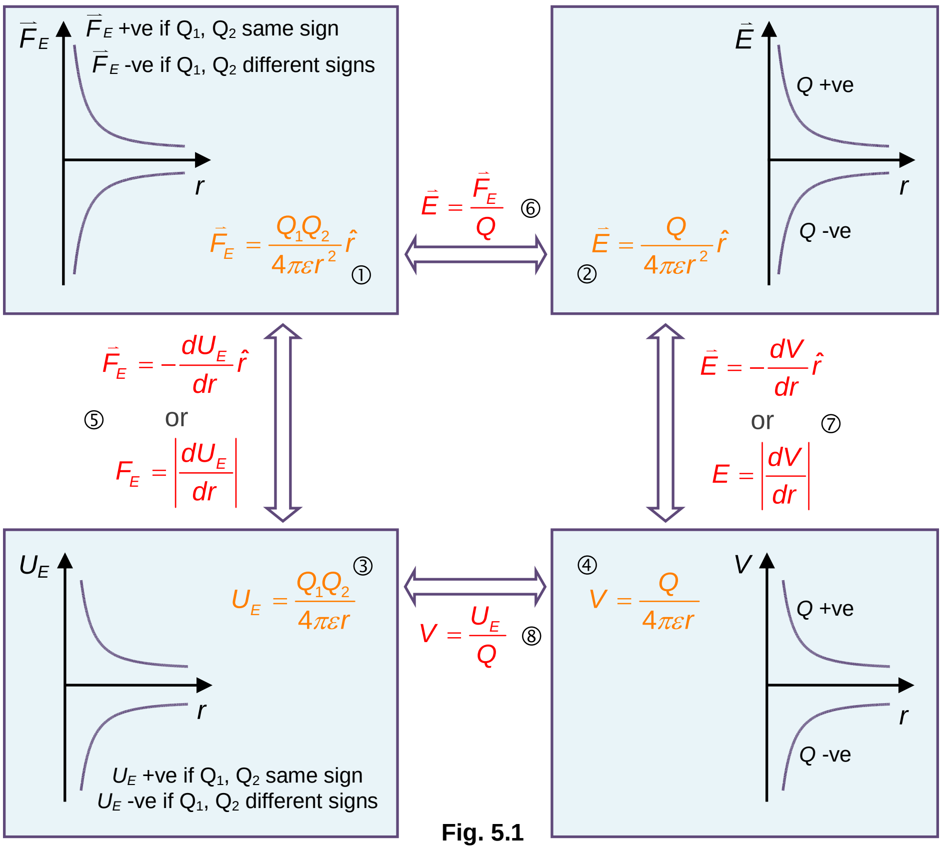


Fig. 5.1

①②③④ are applicable for *point* charges.

⑤⑥⑦⑧ are *general* relationships that arise from the definitions of $\vec{E}$, U_E and V.

Electric force is given by the *potential energy gradient* while field strength is given by the *potential gradient*.

Notation:

$\vec{X}$ denotes a vector, X denotes its magnitude, $\hat{X}$ denotes a unit vector.

①②③④ applicable for *point* charges.

⑤⑥⑦⑧ are general relationships that arise based on the definitions of $\vec{E}$, U_E and V. They link both the equations and graphs in the boxes.

Note that $\vec{F} = -\frac{dU}{dr}\hat{r}$ is in fact a general expression which is valid for gravitational force, electric force, elastic force etc with U being the corresponding type of PE in each case.

Uniform Field

In Fig. 5.2, the zero potential point is taken to be Earth rather than at infinity. This is in fact the norm for electrical circuits. This choice of zero potential does not affect relations ⑤⑥⑦⑧. In contrast ③④ are invalid here because they are based on zero at infinity and only for point charges.

In circuits, the zero potential is by convention taken to be at Earth.

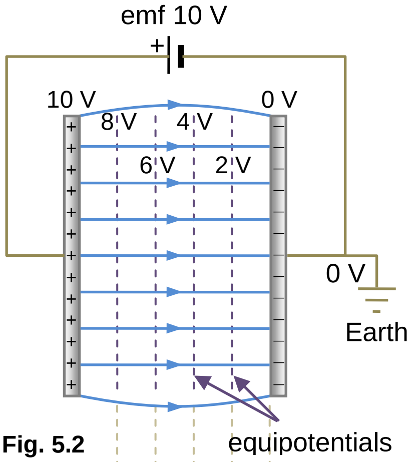


Fig. 5.2

Knowing E field is constant between the plates and $\vec{E} = -\frac{dV}{dr}\hat{r}$, we can deduce that the *V-r* graph must be a straight line (constant potential gradient). –d*V*/d*r* is positive, thus $\vec{E}$ points in the direction of $\hat{r}$ i.e. from left to right. Magnitude wise:

$$E_{uniform} = \left|\frac{\Delta V}{\Delta r}\right|$$

$$E_{uniform} = \left|\frac{\Delta V}{\Delta r}\right|$$

ΔV is the p.d. across the points separated by Δr

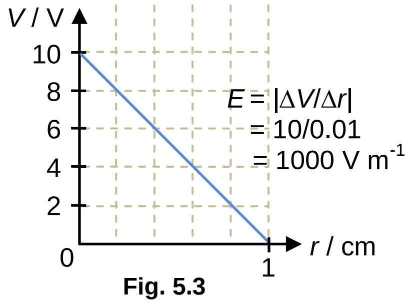


Fig. 5.3

If an electron of charge -*e* is put inside the uniform field, the electric PE between it and the pair of charged plates is given by

$$U_E = QV$$

U_E for the electron at each position is thus shown in Fig. 5.4.

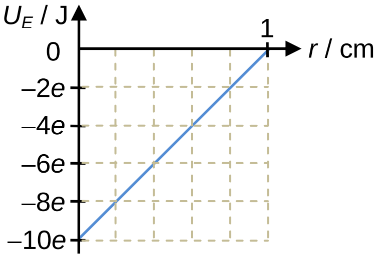


Fig. 5.4

Applying $\vec{F}_E = -\frac{dU_E}{dr}\hat{r}$:

$dU_E/dr = \Delta U_E/\Delta r = 10e/0.01$
$= 1.6 \times 10^{-16}$ N

So $\vec{F}_E = -1.6\times10^{-16}\hat{r}$ i.e. force on electron is 1.6×10^{-16} N pointing opposite to $\hat{r}$ or from right to left which is expected as electron should be attracted by positive plate and repelled by negative plate.

Uniform E Field Vs G Field

From Fig. 5.1 relation ⑧, $V = \frac{U_E}{Q}$, for a charge *Q* moved between two points with a potential difference ΔV, the corresponding change in the electric PE is *always* given by

$$\Delta U_E = Q\Delta V \text{ or } |\Delta U_E| = |Q\Delta V| \quad \text{--- (1)}$$

For *uniform* E field, $E = |\Delta V/\Delta r|$ --- (2)

(1) & (2) $\Rightarrow |\Delta U_E| = |QE\Delta r|$ --- (3)

(3) is the electric counterpart of $\Delta U_G = mg\Delta r$ or *mgh* in the gravitational case. *E* and *g* are counterparts. *q* and *m* are counterparts but an important difference between *q* and *m* is that *q* can either be positive or negative. Also, (3) can be written as $|\Delta U_E| = |F_E\Delta r|$ = magnitude of work done by F_E. Due to all the similarities, the motion of a charge *Q* inside a uniform E field is similar to the motion of a mass in a uniform gravitational field.

Motion of charged particle in a uniform E field is similar to the motion of a mass in a uniform gravitational field i.e. kinematics equations and concepts in projectile motion apply.

Non-uniform Field

An *equipotential* is a line or surface joining all neighbouring points at the same potential. For the charged sphere (Fig. 5.5), the equipotentials are spheres enclosing it.

An *equipotential* is a line or surface joining all neighbouring points at the same potential.

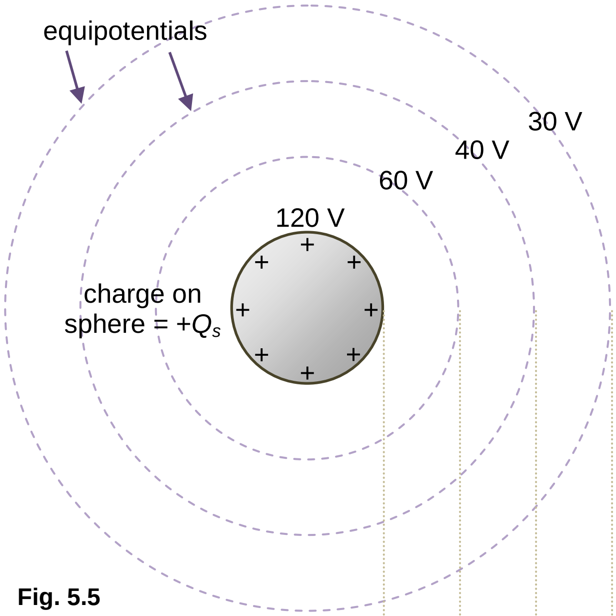


Fig. 5.5

Fig. 5.7 shows the PE between Q and Q_s when charge Q is at different r.

When potentials are plotted against r Fig. 5.6 is obtained.

Fig. 5.6 to 5.9 show how, from V-r graph, the relations ⑤⑥⑦⑧ can be used to obtain other graphs.

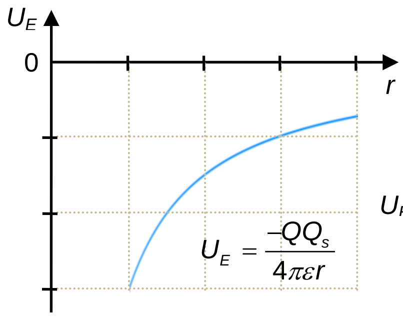


Fig. 5.7

$U_E = -QV$

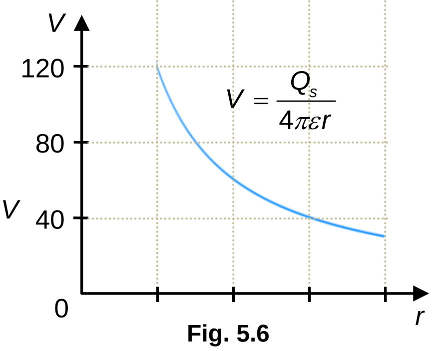


Fig. 5.6

$\vec{F}_E = -\dfrac{dU_E}{dr}\hat{r}$

$\vec{E} = -\dfrac{dV}{dr}\hat{r}$

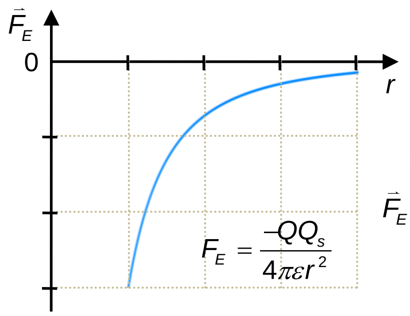


Fig. 5.9

$\vec{F}_E = -Q\vec{E}$

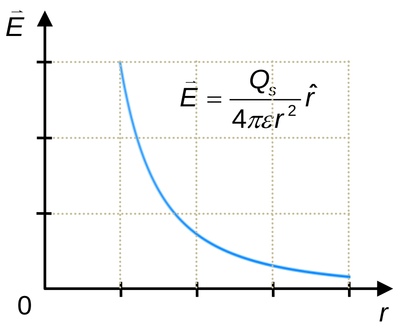


Fig. 5.8

Fig. 5.9 can be obtained by either differentiating the U_E-r graph or simply by product of $-Q$ and $\vec{E}$

Fig. 5.8 can be obtained by differentiating the V-r graph.