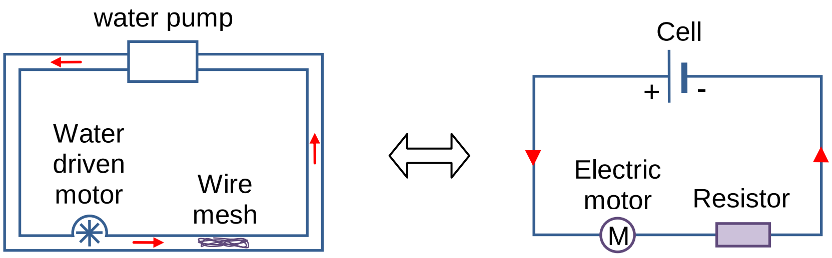
1 Electrical Circuit Model



In steady state: KE imparted by pump to circulating water = KE transferred to motor + KE converted to heat when squeezing through wire mesh obstruction.	In steady state: Electrical PE imparted by cell to circulating electrons = electrical PE transferred to motor + electrical PE converted to heat in resistor.

2 P.d. V = I R

Current *I*

Electrical current is defined as the rate of flow of charge. $I = \frac{dQ}{dt}$

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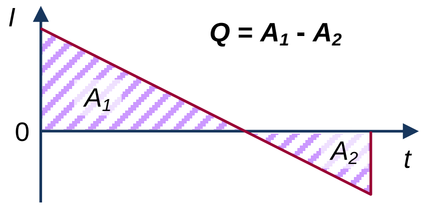
When *I* is constant within time interval Δt, the charge that flowed $Q = I\Delta t$.

For *I* not constant, the total charge that flowed is given by the area of *I*-*t* graph: $Q = A_1 - A_2$



The word 'charge' refers to the *property* of particles such as protons and electrons that is associated with electric field and force just like *mass* is associated with gravitational field and force.

Charge ('unbalanced' charge) is given by $Q = It$ or area under *I*-*t* graph.

As we know, charge can be positive or negative. When we say 'charge on a metal sphere', the 'charge' here refers to the *unbalanced* charge. Hence if proton and electron pairs(i.e. hydrogen atoms) were to go pass a particular point, the net 'charge' flow or current is zero. If someone says 'the charges on the metal sphere', he is probably referring to the *charged particles* on that sphere.

Coulomb is defined as the charge that flowed when a current of *one ampere* is maintained for *one second*.

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Potential difference *V*

In the absence of resistive forces, an object can maintain a constant velocity without an agent applying a force on it. However, whether in the water pipes or wires, there are resistive forces, hence water requires a pressure difference and electrons require a potential difference to maintain the flow.

For an electrical circuit segment with resistance *R*, the larger the resistance, the larger will be the potential difference *V* required to maintain the current. If *R* is constant, then larger *V* is needed for a larger *I* – therefore $V = IR$.

P.d. is needed to drive current through circuit element with resistance: $V = IR$

[Beyond syllabus: in electrical circuits, there are other non-resistive devices such as inductors and capacitors which can link *V* and *I* in different ways. For capacitor, $I = C(dV/dt)$. For inductor coil, $V = L(dI/dt)$.]

3 EMF and P.d.

Definitions of EMF and P.d.

An electrical power source such as a battery, generator and solar cell has a property called *electromotive force*(emf).

> *Emf* is defined as the energy converted from non-electrical to electrical form per unit charge delivered by a source.

> *Potential difference* across a device is defined as the energy converted from electrical to non-electrical form per unit charge

Kirchoff's Voltage Law

These definitions can be linked to formulae $U_E = QV_E$ and $\Delta U_E = Q\Delta V_E$ from the topic of *E Field*. When a charge Q moved through a change in potential ΔV_E, the corresponding change in electrical potential energy(EPE) is ΔU_E. An emf source increases the EPE of the electrons when pumping electrons through it while the resistor decreases their EPE.

As stated in section 1: EPE imparted by cell to circulating electrons = EPE transferred to motor + EPE converted to heat in resistor. Using $\Delta U_E = Q\Delta V_E$, for charge Q to travel one round the circuit, $QE = QV_1 + QV_2$ where E is the emf, V_1 is the p.d. across the motor and V_2 is the p.d. across the resistor.

In general $E = V_1 + V_2 + . . . + V_n$ where there are n devices 'consuming' the electrical energy supplied by the cell. This statement of 'supply = demand' is actually known as Kirchoff's voltage law. It is a statement which says that all energies are accounted for; there is no energy created or destroyed.

Another way to understand Kirchoff's voltage law is that as the charge Q travels around any loop in a circuit, it must encounter changes in potentials ΔV_E which add up to zero: $E - V_1 - V_2 - . . . - V_n = 0$. In other words, when a charge Q goes on any journey and returns to the starting point, it must end up at the same electrical potential. Similarly for a mass that goes on any journey that ends at the starting point must end up at the same gravitational potential as when it started off.

P.d. vs Potentials

Potential is the property *at a point*. Its value depends on which reference position we take to be zero. In the topic *E Field*, we take zero to be at infinity but in electrical circuits, the zero potential point is taken wherever is *earthed*.

Potential difference is literally the difference in potential between two points and the value is independent of where the zero point is. Example: in Fig.3.1 and Fig. 3.2, the zero potential point is different. That only changes the potentials of the various points in the circuit but the p.ds are unchanged.

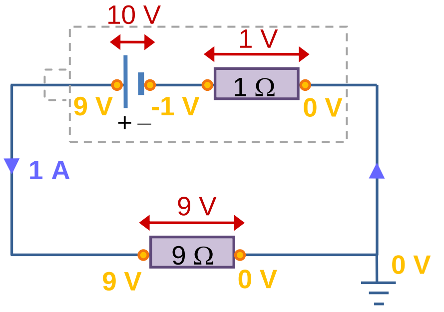


Fig. 3.1

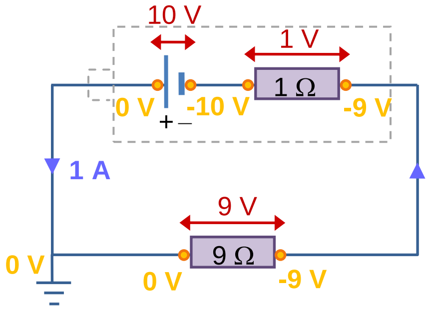


Fig. 3.2

Emf is defined as the energy converted from *non-electrical to electrical* form per unit charge delivered by a source.

Potential difference across a device is defined as the energy converted from *electrical to non-electrical* form per unit charge.

The *volt* is the potential difference between two points that does one joule of work (or result in one joule of energy transfer) when one coulomb flows from one point to the other.

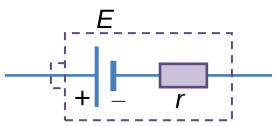
Kirchoff's voltage law:
$E = V_1 + . . . + V_n$
Total emf = total p.d. available in any loop.

The *earthed* point in a circuit is taken to be at zero potential.

Potential values dependent on where zero potential is but p.d. values do not.

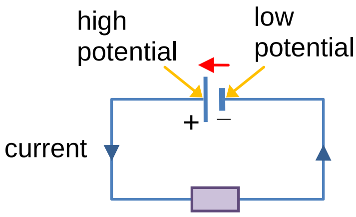
4 Cells, Internal Resistance, Terminal P.d.

A real cell is always considered as made up of an ideal emf source *E* and an internal resistance *r*. Typical values of *r* range from 0.1 Ω to a few Ω.



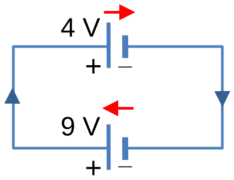
A real cell is made up of
1. ideal emf source *E*
2. internal resistance *r*

The positive terminal of an emf source is *always* at higher potential than its negative terminal.
Outside a cell, *conventional* current *always* flows from higher potential points to lower potential points (resistive circuits, see right).



+ ve terminal of emf source is *always* at higher potential than at -ve terminal.

In *resistive* circuits, *outside cells*, direction of current is always from high to low potential.



However, inside and through a cell, the current may flow *either* way depending on whether other cells are present (see left).

As can be seen in Fig. 3.1, the internal resistance in a real cell takes up a p.d.(1 V) out of the total p.d.(10 V) which is equal to the emf(10 V). This means that the internal resistance produces 1 J of waste heat for every coulomb of charge that passes through it.

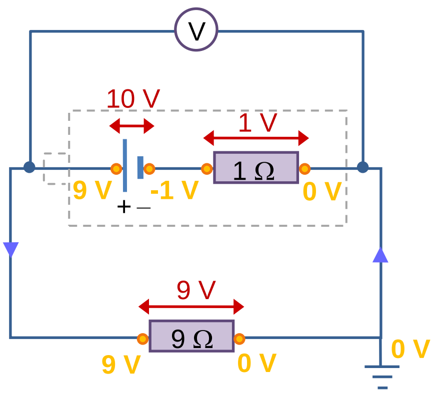
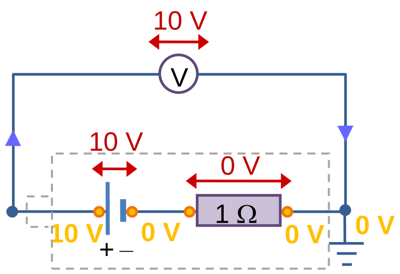


Fig. 3.1

When a voltmeter is connected across the cell, its reading is called *terminal p.d.* and in this case is 9 V. Note that the reading is not the emf of the cell but the net difference in potential across the emf source and the internal resistance. However, the meter's reading equals the p.d. of the external resistance. Here we assume that the meter has infinite resistance such that it does not divert any current away from the original loop and hence it will not change the original potential values.

Terminal p.d. is the p.d. obtained by connecting a voltmeter across the terminals of a cell.

The presence of internal resistance reduces the available p.d.(or terminal p.d.) for the external devices and hence reduces the available output power of the cell. However if no external devices are connected to the cell except the voltmeter, then the terminal p.d. will be equal to the emf:



Assuming a real voltmeter with very high resistance compared to the internal resistance is used. Now, the voltmeter is not only a measuring instrument but is also a crucial component of the circuit. In fact, its high resistance means that it will account for almost all the conversion of EPE to heat i.e. it takes up almost all the total available p.d. which equals the emf.

Internal resistance reduces the available p.d.(or terminal p.d.) for external devices and hence reduces the available output power of the cell.

5 Resistance, Resistivity, Ohm's Law

Resistance of a Conductor

Resistance of a conductor is defined as the ratio of potential difference across it to the current through it. $R = \frac{V}{I}$

Definition of *R*: ratio of *V* to *I*. Not gradient of V-I graph.

Resistance is a property of a conductor and it depends on its dimensions – length and area:

$$R = \frac{\rho L}{A}$$

where ρ is the resistivity
L is the length parallel to the current
A is the cross sectional area perpendicular to current

Resistance depends on dimensions; calculated by $R = \frac{\rho L}{A}$

Resistivity

While resistance is the property of a *conductor*, resistivity is the property of a *material*. It is similar to heat capacity being the property of an object and specific heat capacity being the property of a material. Properties of objects depend on the size while properties of materials are independent of size.

Instead of depending on size, resistivity is dependent on factors such as *temperature* and the number of *charge carriers*(e.g. electrons) per unit volume of conductor. With increased temperature, the atoms in the conductor vibrate more, thereby causing more obstruction(resistance) to the flow of charge carriers. As for charge carrier number per unit volume, it *might* be increased with higher temperature or the addition of impurities depending on the material.

Resistivity as property of a *material* is independent of dimensions but dependent on factors like temperature and charge carrier density.

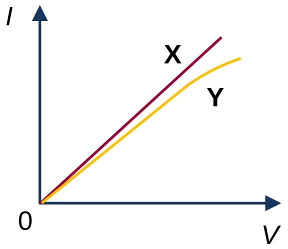
Ohm's Law

The 'law' is not universally applicable, just like Hooke's law.

Ohm's law states that the current through a conductor is proportional to the potential difference across it. $V \propto I$

The constant of proportionality is the resistance. Since there are many conductors whose resistance change with current or p.d, they are said to disobey Ohm's law or *non-ohmic*. Those which obey the law, to varying extent, are called *ohmic*. Even the ohmic ones are usually only ohmic for a limited range of current or p.d.

Ohm's law applies to ohmic conductors where the current through it is proportional to the p.d. across it.

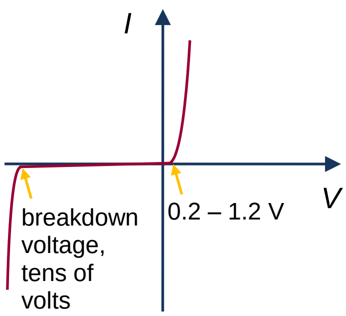


For a metallic conductor at *constant temperature*, it is *ohmic* and the *I*-*V* graph is proportional (**X**).

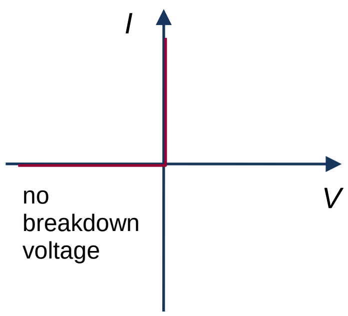
If no effort is made to keep temperature constant as *V* and *I* increases and the conductor heats up, its resistance will increase (**Y**).

Metallic conductors at *constant temperature* are ohmic but not ohmic if allowed to heat up significantly with larger *V* and *I*.

A semiconductor diode is non-ohmic and its *I*-*V* graph is as follows:



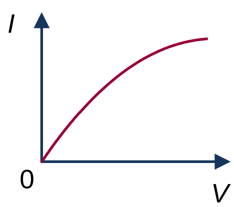
A real diode



An ideal diode

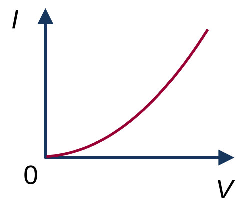
Semiconductor diodes are non-ohmic and their *I*-*V* graphs are shown to the left.

The filament of a lamp is typically made of very thin tungsten. Tungsten is a metal but because of the filament's thinness, its resistance is high enough for it to get hot easily and radiate light for small increase in voltage. With higher temperature comes higher resistivity and even greater resistance.



Filament is non-ohmic as its resistance starts high and heats up with bigger p.d, leading to increased resistivity and resistance.

A common thermistor is made from semiconductor whose resistance *decreases* easily with temperature. When the p.d. is increased modestly, it heats up. The higher temperature leads to the release of electrons which were previously bound and immobile. The higher number of charge carriers result in smaller resistivity and thus resistance. This effect outweighs the effect of increased vibration of the atoms.



The thermistor is non-ohmic. Its resistance decreases with temperature.

6 Power P = I V

Formulae

From definition of p.d. $V = \frac{En}{Q}$ or $\frac{Energy}{Charge}$

$$V = \frac{En/t}{Q/t}$$

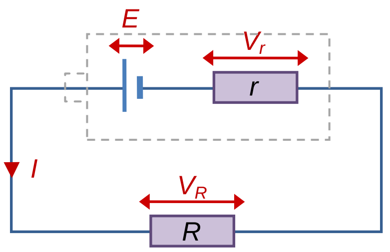
$$V = \frac{P}{I}$$

Hence, $P = IV$

For resistive conductors, $V = IR$, hence, $P = I^2R$ & $P = V^2/R$

$P = IV$ is derived from definition of p.d.
Using $V = IR$, P also $= I^2R$ & V^2/R

Power in a circuit



From Kirchoff's Voltage Law,

total emf = total p.d.

$E = V_R + V_r$

$IE = IV_R + IV_r = I^2R + I^2r$

which means total power provided by emf source = power dissipated in R + power dissipated in r

Emf = total p.d. implies power from emf source = total power 'used' by components in circuit

In general, when two resistances r and R are in series (just like above), the power dissipated to them is in the ratio of their resistances. However, does that mean that if we want to achieve maximum power to R, we should make R as big as possible?

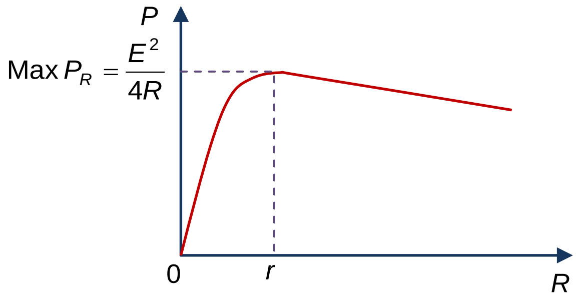
$P_R = I^2R$ and using $E = IR + Ir$ to find $I = \left(\frac{E}{R+r}\right)$

$P_R = \left(\frac{E}{R+r}\right)^2 R$ *where E and r are fixed.*

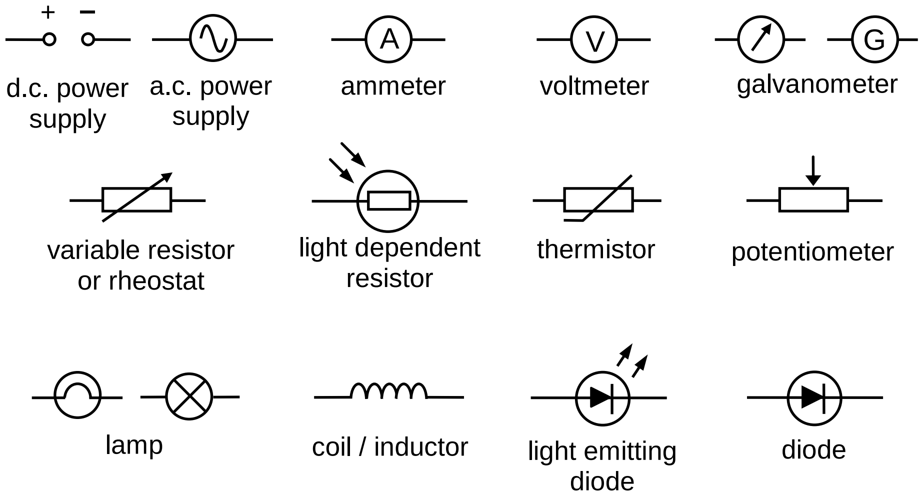
R for which P_R is maximum can be found by $dP_R/dR = 0$ (not needed in syllabus) and it can be shown that *maximum power to the external resistance is achieved when R is adjusted to match r* – a general result.

As R is increased from 0, an increasing *percentage* of the total power goes to R, but it also leads to a smaller current which decreases the total power(IE) available for sharing and eventually causing P_R to decease.

For a cell with internal resistance r connected to variable external resistance R, the *maximum power to R is when R = r*



7 Circuit Components and Symbols



The *light dependent resistor* (LDR) is a resistor whose resistance drops when exposed to brighter light.

8 Parallel and Series Circuits

How do you identify series and parallel components?

Series

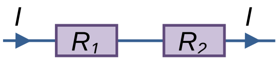


Fig. 8.1

Two resistors in series must have the *same current* (Fig. 8.1), though two resistors with same current may not be in series.

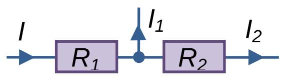


Fig. 8.2

There *cannot be any net current flowing out or into the connection between them* (Fig. 8.2).

Resistors in series
1. must have the same current and
2. there must not be net flow of current in or out of the connection between them.

Parallel

Two resistors in parallel must have the *same p.d.* (Fig. 8.3), though two resistors with same p.d. may not be parallel.

The resistors must have a *common connection* **L** at one potential and another *common connection* **R** at another potential.

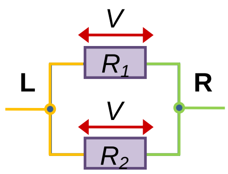
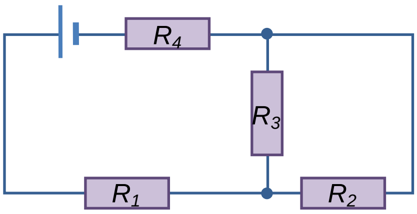


Fig. 8.3

Parallel resistors
1. must have the same p.d. and
2. share common connections on both ends.

Effective Resistance

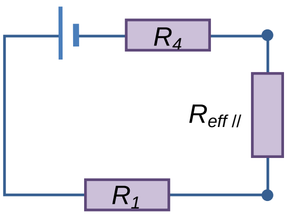
An *effective resistance* is a resistance which replaces two or more resistances without changing the p.d.s and currents in the rest of the circuit.



R_2 and R_3 are parallel and their effective resistance $R_{eff\,//}$ is given by:

$$\frac{1}{R_{eff\,//}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eff\,//} = \frac{R_2 R_3}{R_2 + R_3} \quad \text{(product} \div \text{sum)}$$



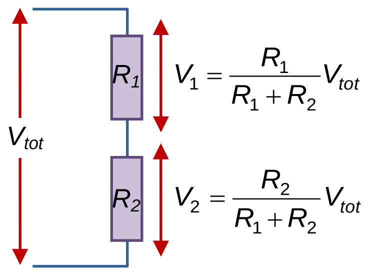
Replacing R_2 and R_3 by $R_{eff\,//}$ will not change the p.ds across R_1 and R_4 or the current through them.

R_1 and R_4 are in series and their effective resistance $R_{eff\,--} = R_1 + R_4$

Effective resistance is one which replaces two or more resistances without changing the p.d.s and currents in the rest of the circuit.

9 Potential Divider and Balancing P.ds

Potential Divider



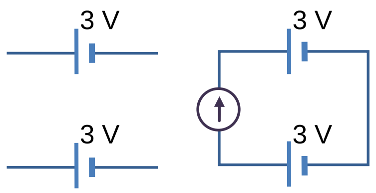
Given two resistors in series, each resistor's share of the total p.d. is in direct proportion to its resistance.

Also, the ratio of their p.ds is the ratio of their resistances.

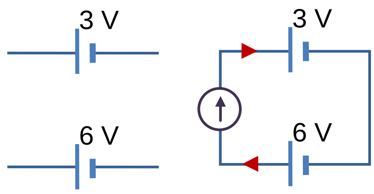
Acquiring an intuitive sense of the above two statements is very useful for quickly assessing a given circuit.

Given two resistors in series, each resistor's share of the total p.d. is in direct proportion to its resistance.

Balancing of P.ds

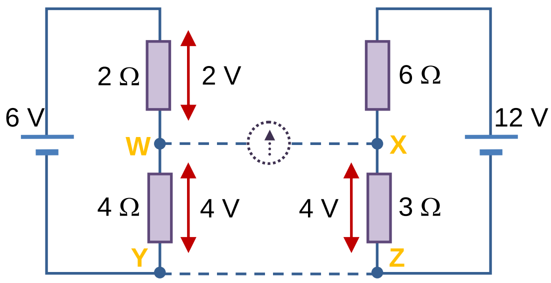


Two opposing p.ds of the same magnitude will give no deflection in the galvanometer when connected.



Two opposing p.ds of different magnitude will give a deflection in the galvanometer when connected.

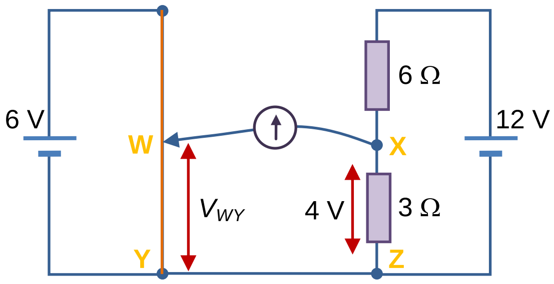
The p.ds do not have to be from cells. They can be from potential dividers:



If the two loops were connected as shown, **W** to **X** and **Y** to **Z**, there will be no current in **WX** and **YZ**.

The two loops are effectively operating independently.

A potential divider can also be made from a resistance wire(potentiometer):

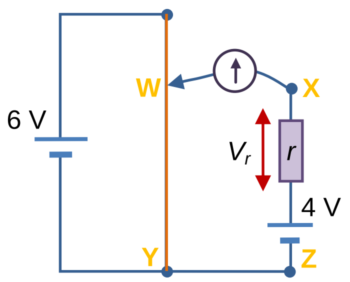


If the jockey is tapped on **W** and there is no deflection, it means p.d. $V_{WY} = V_{XZ}$.

If the 6 V cell is ideal and the resistance wire is 100 cm, then the balance length **WY** must be (4/6)100 or 66.7 cm.

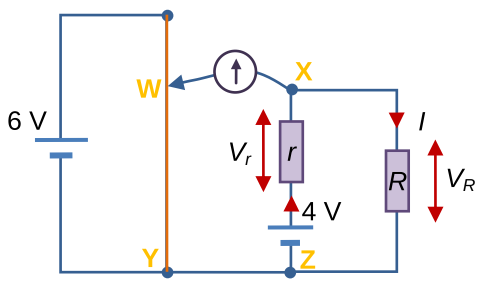
If two *opposing* p.ds balance, there is no current flow. The p.ds can come from cells or potential dividers or potentiometer

Two more advanced situations below:



Note, when $V_{WY} = V_{XZ}$:
V_{XZ} = emf of 4 V cell.

This is because $V_r = Ir$ & $I = 0$ in **XZ** $\therefore V_r = 0$.



Note, when $V_{WY} = V_{XZ}$, $V_{XZ} = V_R$.

The rightmost loop is effectively independent from the leftmost loop as there is no current flow between them.